The Third Pan-Pacific International Conference on Topology and Applications

November 8–13, 2019, Xiangyu Hotel, Chengdu
People's Republic of China

Speakers and Abstracts

Hosted by the School of Mathematics, Sichuan University
People's Republic of China

The Pan-Pacific International Conference on Topology and Applications (PPICTA) is a biennial conference that brings together researchers from topology and various application areas to discuss recent key achievements, new problems and future directions for research, and also helps promoting the academic exchange and the friendship among researchers on this area and related topics.

The first two conferences were held in 2015 at Minnan Normal University and in 2017 at Pusan National University, respectively.

The third Pan-Pacific International Conference on Topology and Applications was hosted by Sichuan University and held at Xiangyu Hotel, Chengdu, People's Republic of China, November 8-13, 2019. The conference had the following **7** sessions:
- Algebraic Topology
- General and Set-theoretic Topology
- Geometric Topology
- Low-dimensional Topology
- Order, Topology and Theoretic Computer Science
- Set-Theory
- Topology and Dynamical Systems

with **7** plenary talks, **45** invited talks and **93** contributed talks.

International Organizing Committee

Jiling Cao (Chair), Sergey Antonyan, Pratulananda Das, Fuquan Fang, Salvador Garcia-Ferreira, Sang-Eon Han, Yasunao Hattori, Boju Jiang, Sang Youl Lee, Anmin Li, Fucai Lin, Shou Lin, Maokang Luo, Seiichi Kamada, Manuel Sanchis, Dmitri Shakhmatov, Mikhail Tkachenko, Boaz Tsaban, Shicheng Wang, Zhongqiang Yang

Local Organizing Committee

Weinian Zhang (Chair), Hui Kou (Co-deputy chair), Shuguo Zhang (Co-deputy chair), Bohui Chen, Dexue Zhang

Session Organizers

Algebraic Topology

Daniel Juan-Pineda, UNAM, Mexico
Daisuke Kishimoto, Kyoto University, Japan
Yongjin Song, Inha University, Republic of Korea
Bai-Ling Wang, ANU, Australia

General and Set-theoretic Topology*

Jiling Cao, Auckland University of Technology, New Zealand
Salvador Garcia-Ferreira, UNAM, Mexico
Yasunao Hattori, Shimane University, Japan
Alejandro Illanes, UNAM, Mexico

**Dedicated to Prof Salvador Garcia-Ferreira and Prof Valentin Gutev for their 60th birthday.*

Geometric Topology

Bohui Chen, Sichuan University, China
Sergey A. Antonyan, UNAM, Mexico
Bin Zhang, Sichuan University, China

Low-dimensional Topology

Mario Eudave-Munoz, UNAM, Mexico
Seiichi Kamada, Osaka University, Japan
Sang Youl Lee, Pusan National University, Republic of Korea
Weiping Li, Southwest Jiaotong University, China

Order, Topology and Theoretic Computer Science

Sang-Eon Han, Chonbuk National University, Republic of Korea
Hui Kou, Sichuan University, China
Dexue Zhang, Sichuan University, China

Set Theory

Sakae Fuchino, Kobe University, Japan
Su Gao, University of North Texas, USA
Shuguo Zhang,Sichuan University, China

Topology and Dynamical Systems

Hisao Kato, University of Tsukuba, Japan
En Hui Shi, Soochow University, China
Weinian Zhang, Sichuan University, China

Plenary Speakers

Jörg Brendle, Kobe University, Japan

Dirk Hofmann, University of Aveiro, Portugal

Rita Jimenez, Universidad Nacional Autónoma de México, Mexico

Kaoru Ono, Kyoto University, Japan

Shicheng Wang, Peking University, China

Takamitsu Yamauchi, Ehime University, Japan

Xiangdong Ye, University of Science and Technology of China, China

Invited and Contributed Speakers

Algebraic Topology

Invited speakers

Noé Bárcenas, CCM-UNAM, Mexico
Suyoung Choi, Ajou University, Republic of Korea
Teng Huang, University of Science and Technology of China, China
Daisuke Kishimoto, Kyoto University, Japan
Katsuhiko Kuribayashi, Shinshu University, Japan
Takahiro Matsushita, University of the Ryukyus, Japan
Haozhi Zeng, Huazhong University of Science and Technology, China

Contributed speakers

Toshiyuki Akita, Hokkaido University, Japan
Ho Won Choi, Korea University, Republic of Korea
Chengyong Du, Sichuan Normal University, China
Ruizhi Huang, Chinese Academy of Sciences, China
Zhenxi Huang, Sichuan University, China
Norio Iwase, Kyushu University, Japan
Daniel Juan Pineda, UNAM, Mexico
Xiaobin Li, Southwest Jiaotong University, China
Atsushi Yamaguchi, Osaka Prefecture University, Japan
Min Yan, Hong Kong University of Science and Technology, China
Jianqiang Yang, Honghe University, China

General and Set-theoretic Topology*

Invited speakers

Salvador Garcia-Ferreira, UNAM, Mexico
Valentin Gutev, University of Malta, Malta
Wei He, Nanjing Normal University, China
Verónica Martínez de la Vega, UNAM, México
Manuel Sanchis, Universitat Jaume I, Spain
Dmitri Shakhmatov, Ehime University, Japan
Mikhail Tkachenko, Universidad Autónoma Metropolitana, Mexico

Contributed speakers

Ibtesam Alshammari, University of Hafr Al Batin, Saudi Arabia
Angelo Bella, University of Catania, Italy
Manoj Bhardwaj, University of Delhi, India
Zhangyong Cai, Nanning Normal University, China
Jiling Cao, Auckland University of Technology, New Zealand
Víctor Hugo Yañez, Ehime University, Japan
Alejandro Illanes, UNAM, México
Shou Lin, Ningde Normal University, China
Jorge M. Martínez-Montejano, UNAM, México
Lei Mou, Capital Normal University, China
Jimmy A. Naranjo-Murillo, UNAM, México

Liangxue Peng, Beijing University of Technology, China
Iván Sánchez
Rongxin Shen, Taizhou University, China
Ángel Tamariz-Mascarúa, UNAM, Mexico
Huan Wang, Beijing university of technology, China
Li-Hong Xie, Wuyi University, China
Yukinobu Yajima, Kanagawa University, Japan
Hanbiao Yang, Wuyi University, China
Cenobio Yescas-Aparicio, UNAM, México

**Dedicated to Prof Salvador Garcia-Ferreira and Prof Valentin Gutev for their 60th birthday.*

Geometric Topology

Invited speakers

Sergey Antonyan, UNAM, Mexico
Natalia Jonard-Perez, UNAM, Mexico
Katsuhisa Koshino, Kanagawa University, Japan
Li Sheng, Sichuan University, China
Bailing Wang, The Australian National University, Australia
Ming Xu, Capital Normal University, China

Contributed speakers

Niufa Fang, Nankai University, China
Xiao Li, Southwest University, China
Li-Jie Sun
Xinxing Tang, Tsinghua University, China
Song Yang, Tianjin University, China
Xiangdong Yang, Chongqing University, China
Baocheng Zhu, Hubei Minzu University, China

Low-dimensional Topology

Invited speakers

Bruno A. Cisneros de la Cruz, UNAM, Mexico
Akio Kawauchi, Osaka City University, Japan
Yi Liu, BICMR, China
Jongil Park, Seoul National University, Republic of Korea
Jesus Rodriguez-Viorato, CIMAT, Guanajuato, Mexico

Contributed speakers

Sukuse Abe, Osaka City University, Japan
Hirotaka Akiyoshi, Osaka City University, Japan
Yongju Bae, Kyungpook National University, Republic of Korea
Benjamin Bode, Osaka University, Japan
Ioannis Diamantis, China Agricultural University, China
Mario Eudave-Muñoz, UNAM, Mexico
José Frías, UNAM, Mexico

Hiroshi Goda, Tokyo University of Agriculture and Technology, Japan
Megumi Hashizume, Nara University of Education and OCAMI, Japan
Atsushi Ishii, University of Tsukuba, Japan
Noboru Ito, The University of Tokyo, Japan
Naoko Kamada, Nagoya City Univeristy, Japan
Seiichi Kamada, Osaka Univeristy, Japan
Taizo Kanenobu, Osaka City University, Japan
Kengo Kawamura, Osaka City University, Japan
Sang Youl Lee, Pusan National University, Republic of Korea
Fengling Li, Dalian University of Technology, China
Chan Palomo Luis Celso, Universidad Autónoma de Yucatán, México
Shosaku Matsuzaki, Takushoku University, Japan
Tomo Murao, University of Tsukuba, Japan
Takefumi Nosaka, Tokyo Institute of Technology, Japan
Makoto Ozawa, Komazawa University, Japan
Ruifeng Qiu, East China Normal University, China
Alexander Stoimenow, Gwangju Institute of Science and Technology, Republic of Korea
Hideo Takioka, Kyoto University, Japan
Masakazu Teragaito, Hiroshima University, Japan
Jun Ueki, Tokyo Denki University, Japan
Yoshiro Yaguchi, National Institute of Technology, Gunma College, Japan
Jingling Yang, The Chinese University of Hong Kong, China
Wenyuan Yang, Peking University, China
Shengkui Ye, Xi'an Jiaotong-Liverpool University, China
Seokbeom Yoon, Korean Institute for Advanced Study, Republic of Korea
Ying Zhang, Soochow University, China
Yanqing Zou, Dalian Minzu University, China

Order, Topology and Theoretic Computer Science

Invited speakers

Soichiro Fujii, Kyoto University, Japan
Hongliang Lai, Sichuan University, China
Jing Lu, Xi'an Jiaotong University, China
Zhenchao Lyu, Sichuan University, China
Lili Shen, Sichuan University, China
Alexander Šostak, University of Latvia, Latvia
Xiaoquan Xu, Minnan Normal University, China

Contributed speakers

Shen Chong, Beijing Institute of Technology, China
Sang-Eon Han, Chonbuk National University, Republic of Korea
Wonse Kim, Seoul National University, Republic of Korea
Alexander Šostak, University of Latvia, Latvia
Longchun Wang, Hunan University, China
Dexue Zhang, Sichuan University, China

Set Theory

Invited speakers

David Asperó, University of East Anglia, UK
Joan Bagaria, University of Barcelona, Spain
Aleksander Blaszczyk, University of Silesia, Poland
Longyun Ding, Nankai University, China
Sakaé Fuchino, Kobe University, Japan
Jialiang He, Sichuan University, China
André Ottenbreit Maschio Rodrigues, Kobe University, Japan
Yinhe Peng, CAS, China
Franklin Tall, University of Toronto, Cannada
Toshimichi Usuba, Waseda University, Japan

Contributed speakers

Zuoheng Li, Sichuan University, China
Sumit Singh, University of Delhi, India
Jaroslav Šupina, University of P.J. Safarik, Slovakia

Topology and Dynamical Systems

Invited speakers

Zeng Lian, Sichuan University, China
Andrei Malyutin, St. Petersburg State University, Russia
Peiyong Zhu, University of Electronic Science and Technology of China, China

Contributed speakers

Jianyu Chen, Soochow University, China
Hisao Kato, University of Tsukuba, Japan
Jian Li, Shantou University, China
Yanlin Li, Hangzhou Normal University, China
Gang Liao, Soochow University, China
Y. Rodríguez-López, Universidade Federal de Minas Gerais, Brazil
Enhui Shi, Soochow University, China
M. Shlossberg, Shamoon College of Engineering, Israel
Benjamin Vejnar, Charles University, Czech Republic
Suhua Wang, Jiangsu University of Science and Technology, China
Wenfei Xi, Nanjing University of Finance & Economics, China
Hui Xu, Soochow University, China

Abstracts of Plenary Talks

Cardinal invariants and forcing theory

Jörg Brendle

Kobe University, Japan
Email: brendle@kobe-u.ac.jp

Cardinal invariants of the continuum are cardinal numbers describing the combinatorial structure of the real numbers (that is, the Cantor space 2^ω or the Baire space ω^ω) and typically taking values between the first uncountable cardinal $\aleph_1$ and the cardinality of the continuum $\mathfrak{c}$. An example is the unbounding number $\mathfrak{b}$, the smallest size of a family of functions in ω^ω unbounded in the eventual dominating ordering. Such cardinal invariants have many applications, in particular in general topology, but also in other areas of pure mathematics.

While cardinal invariants of the continuum have been investigated intensively for decades, more recently people have started to look at the higher Cantor space 2^κ and the higher Baire space κ^κ, where κ is an uncountable regular cardinal, and redefined analogous cardinal numbers, called *higher cardinal invariants*, in this context. Many results known for ω carry over to κ, in particular in the case when κ is a large cardinal. However, there are also several interesting differences between the classical case and higher cardinal invariants.

I will give a survey on recent results and open problems on cardinal invariants, both for the classical case and for higher invariants. Focus will be on independence proofs and the connection with forcing theory.

Enriched Priestley spaces

Dirk Hofmann

University of Aveiro, Portugal
Email: dirk@ua.pt

During the past decades, Lawvere's 1973 [20] interpretation of a metric spaces as a category enriched over the extended real half-line $[0, \infty]$ has experienced wide-spread interest amongst researchers working in computer science and in general topology. His revolutionary idea was not only to regard individual metric spaces as categories enriched in a quantale of non-negative real numbers; Lawvere also showed that categorical notions and results translate into significant ones about metric spaces "so that enriched category theory can suggest new directions of research in metric space theory and conversely". This observation has motivated much work on the reconciliation of order, metric and category theory; to give an impression, we mention the work on

- "metric domain theory" presented in [7, 5, 28] and – using approach spaces (a metric version of topological spaces) – in [29, 8, 21];
- probabilistic and partial metric spaces [6, 13, 17];
- complete distributivity and algebraicity [26, 25, 3];

- generalising classical "order-theoretic" duality results to the metric and quantale-enriched context [4, 16, 2, 9, 30, 12];
- extending Nachbin's notion of ordered topological space [22] to metric spaces and further to enriched categories [27, 10, 14];
- monad and quantale enriched categories [15, 19, 18].

The main motivation for this talk stems from our recent recent study of Stone-type dualities [12], where we extended the context from order structures to metric structures and in particular went from ordered compact Hausdorff spaces to metric compact Hausdorff spaces. We recall that Priestley spaces [23, 24], the duals of distributive lattices, are by definition those partially ordered compact spaces X where the cone

$$(f\colon X \to 2)_f$$

of morphisms into the two-element space is point-separating and initial. Passing to the metric setting, this led naturally to the notion of *metric Priestley space*: those metric compact Hausdorff spaces X where the cone

$$(f\colon X \to [0,1]^{\mathrm{op}})_f$$

of all morphisms into the unit interval is point-separating and initial. We note that, in the metric setting, every partially ordered compact space is Priestley, and so is every classic compact metric space. Inspired by classic results about compact Hausdorff spaces and more recent results about partially ordered compact spaces [11, 1], in this talk we develop duality theory for metric Priestley spaces and in particular investigate the algebraic character of the dual of the category of quantale-enriched Priestley spaces and morphisms.

2010 MSC: Primary 18B35; Secondary 54B30.

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2010 MSC: Primary 03E17; Secondary 03E35.

Floer cohomology and covering spaces

Kaoru Ono

Kyoto University, Japan
Email:ono@kurims.kyoto-u.ac.jp

Some observations on Floer cohomology and covering spaces will be presented. One is about a lower bound for the number of fixed points of Hamiltonian diffeomorphisms in presense of fundamental group. Another is about Floer cohomology for symplectic isotopies and give a slight improvement of my old result on the number of fixed points of the time-one map of symplectic isotopies. The former is based on a joint work with A. Pajitnov and the latter is partly based on joint work with H.-V. Le.

2010 MSC: Primary 53D40; Secondary 37J10.

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Stability patterns in algebra and topology

Rita Jiménez Rolland

UNAM, México
Email: rita@im.unam.mx

In this talk we will consider configuration spaces of n points in a manifold and we will survey how the topology of these spaces changes as the parameter n grows. We will describe a framework that allows to find and predict patterns of algebraic invariants of these and other families of spaces that appear naturally in algebra and topology.

2010 MSC: Primary 55R80; Secondary 20C30; 57T05; 32G15.

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Connectedness properties of the Higson corona of the half line

Takamitsu Yamauchi

Ehime University, Japan
Email: yamauchi.takamitsu.ts@ehime-u.ac.jp

The Higson corona of a proper metric space is a compact Hausdorff space defined as the remainder (boundary) of the Higson compactification. This notion was introduced in coarse geometry ([3]) and it is invariant under coarse equivalences. Iwamoto and Tomoyasu [2] proved that the Higson corona of the half line $\{x \in \mathbb{R} : x \geq 0\}$ with the usual metric is an indecomposable continuum, that is, a compact connected Hausdorff space that cannot be written as the union of two proper compact connected subspaces. This theorem can be regarded as an analogue of Bellamy's theorem [1] on the Čech-Stone remainder of the half line. In this talk, reviewing fundamental results on Higson coronas, we consider further properties on connectedness of the Higson corona of the half line.

2010 MSC: Primary 54D35; Secondary 54F15.

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Embedding the symmetries of surfaces and graphs to those of 3-sphere

Shicheng Wang

Peking University, China
Email: wangsc@math.pku.edu.cn

This is a talk with many intuitive examples and figures.

We consider finite group G acting on the surface F_g of genus g that can be extended to the G action on the 3-sphere S^3, for some embedding $F_g \to S^3$.

The fist half will be survey on the maximum order problems, which are published work jointly with C. Wang, B. Zimmermann, Y. Zhang. See

1. Embedding surfaces into S3 with maximum symmetry. Groups Geom. Dyn. 9 (2015), no. 4, 1001–1045.

2. Graphs in the 3-sphere with maximum symmetry. Discrete Comput. Geom. 59 (2018) no. 2, 331–362.

The next half will discuss: If a periodic map on surface extends to a homeomorphism of S^3, can it extends to a periodical map of S^3 (jointly with Y. Ni and C. Wang)? Can we say some thing if S^3 is replaced by S^4 or S^5 (jointly with N. Wang)?

Topology and topological sequence entropy

L. Snoha, Xiangdong Ye and R. Zhang

University of Science and Technology of China, China
Email: yexd@ustc.edu.cn

Let X be a compact metric space and $T : X \to X$ be continuous. Let $h^*(T)$ be the supremum of topological sequence entropies of T over all subsequences of $\mathbb{Z}_+$ and $S(X)$ be the set of the values $h^*(T)$ for all continuous maps T on X. It is known that $\{0\} \subseteq S(X) \subseteq \{0, \log 2, \log 3, \ldots\} \cup \{\infty\}$ by a result of Huang and Ye proved by combinatorial tools.

We completely solve the problem of finding all possibilities for $S(X)$ by showing that in fact for every set $\{0\} \subseteq A \subseteq \{0, \log 2, \log 3, \ldots\} \cup \{\infty\}$ there exists a one-dimensional continuum X_A with $S(X_A) = A$. In the construction of X_A we use **Cook continua**. This is apparently the first application of these very rigid continua in dynamics.

We further show that the same result is true if one considers only homeomorphisms rather than continuous maps, except the case when A is infinite and $\infty \notin A$ which remains open. The problem for group actions is also addressed but in full generality it remains open

2010 MSC: Primary 54H20; Secondary 37A35.

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Abstracts in Algebraic Topology

On the Borel conjecture for Alexandrov spaces

Noé Bárcenas and Jesús Núñez Zimbrón

CCM-UNAM, Mexico
Email: barcenas@matmor.unam.mx

The Borel Conjecture for aspherical manifolds states that a map inducing isomorphism of fundamental groups of aspherical manifolds is homotopic to a homemomrphism. Using methods of large scale geometry, specifically Gromov-Hausdorff convergence, thurston geometrization, recent advances of classification programs in Alexandrov Geometry and group cohomology, we are able to formulate a Version of the Borel conjecture and prove it for a broad family of examples.

2010 MSC: Primary 53C23; Secondary 57S15.

Topology of real toric manifolds and its application

Suyoung Choi

Ajou University, Republic of Korea
Email: schoi@ajou.ac.kr

Two main objects of toric topology are a toric manifold and its real locus, called a real toric manifold. One interesting open question is the so-called *toric cohomological rigidity problem* which asks whether the cohomology ring of a toric manifold determines its topological type or not.

In the lecture, I introduce recent achievements on topology of real toric manifolds, and its applications to the toric cohomological rigidity problem. This talk is based on the joint works with Hanchul Park and Li Cai.

2010 MSC: Primary 57N65;

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Moduli space of Kapustin-Witten equations on a closed 4-manifold

Teng Huang

University of Science and Technology of China, China
Email: htmath@ustc.edu.cn

In this talk, we first recall the Kapustin-Witten equations on a four-dimensional manifold which were introduced by Kapustin and Witten. The motivation is from the viewpoint of N = 4 super Yang-Mills theory in four dimensions to study the geometric Langlands program. By a compactness theorem due to Taubes, we will prove that if (A, ϕ) is a smooth solution of KW equations and the connection A is closed to a generic ASD connection, then (A, ϕ) must be a trivial solution. We also prove that the moduli space of the solutions of KW equations is non-connected if the connections on the compactification of moduli space of ASD connections are all generic. At last, we extend the results for the KW equations to other equations on gauge theory such as the Hitchin-Simpson equations and Vafa-Witten equations.

2010 MSC: Primary 58E15; Secondary 81T13

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Higher homotopy associativity in the Harris decomposition of Lie groups

Daisuke Kishimoto and Toshiyuki Miyauchi

Kyoto University, Japan
Email: kishi@math.kyoto-u.ac.jp

For certain pairs of Lie groups (G, H) and primes p such as $(SU(2n), Sp(n))$ and $p \geq 3$, Harris showed in 1962 that, in modern words, the fibration $H \to G \to G/H$ splits p-locally such that there is a p-local homotopy equivalence

$$G \simeq_{(p)} H \times G/H.$$

We show how much this homotopy decomposition respects the group structures of G and H in view of higher homotopy associativity.

This is joint work with Toshiyuki Miyauchi.

2010 MSC: Primary 55P60; Secondary 55P35.

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The non-triviality of the labeled open-closed TQFT associated with string topology for classifying spaces

Katsuhiko Kuribayashi

Shinshu University, Japan
Email: kuri@math.shinshu-u.ac.jp

String topology introduced by Chas and Sullivan [1] gives fruitful structures to the loop homology of orientable closed manifolds, more general Gorenstein spaces [3]. In particular, a result due to Chataur and Menichi [2] asserts that the loop homology of the classifying space of a Lie group is endowed with the structure of a 2-dimensional topological quantum field theory (TQFT). Guldberg [4] has proved that such a structure is generalized to that of a labeled open-closed TQFT. However, there are few calculations of labeled cobordism operations in the theory. In this talk, via an explicit calculation, we explain the non-triviality of a whistle cobordism operation with labels in the set of maximal closed subgroups of the given Lie group. It turns out that the open TQFT and closed one are not separated in general. This talk is based on the paper [5].

2010 MSC: Primary 55P50; Secondary 81T40, 55R35.

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Relative phantom maps

Kouyemon Iriye, Daisuke Kishimoto, and Takahiro Matsushita

University of the Ryukyus, Japan
Email: mtst@sci.u-ryukyu.ac.jp

A phantom map from a CW-complex X to a space Y is a map $f\colon X \to Y$ such that the restriction of f to every finite dimensional skeleton X^n is null-homotopic. We generalize phantom maps to as follows: Let X be a CW-complex and $\varphi\colon B \to Z$ be a map between spaces.

A relative phantom map from X to φ is a map $f\colon X \to Y$ such that $f|_{X^n}$ has a lift, up to homotopy. This notion is inspired by de Bruijn-Erdős' theorem in graph theory.

In this talk, we introduce relative phantom maps and talk about their properties. In particular, we partially generalize a result of McGibbon and Roitberg [3] concerning phantom maps and rational homotopy.

2010 MSC: Primary 55P99; Secondary 55P62.

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On Fano and weak Fano regular Hessenberg varieties

Haozhi Zeng

Huazhong University of Science and Technology, People's Republic of China
Email: zenghaozhi@hust.edu.cn

Regular Hessenberg varieties are a family of subvarieties of the full flag variety G/B. This family contains the full flag variety, Peterson variety and perutohedral variety. In this talk, we discuss the Fano and weak Fano regular semisimple Hessenberg varieties in type A. We give a sufficient and necessary condtion for a regular semisimple Hessenberg variety to be Fano and weak Fano. This is joint work with Hiraku Abe and Naoki Fujita.

2010 MSC: Primary 57N65;

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On the cohomology of Coxeter groups and related groups

Toshiyuki Akita

Hokkaido University, Japan
Email: akita@math.sci.hokudai.ac.jp

In this talk, I will discuss my results on the cohomology of Coxeter groups and their alternating subgroups, Artin groups, and the adjoint groups of Coxeter quandles. Part of this talk is based on joint work with Ye Liu.

2010 MSC: Primary 20F55, 20J06; Secondary 55N91.

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Self closeness numbers and Self lengths of certain monoids of self maps

Ho Won Choi and Kee Young Lee

Institute of Natural Science, Korea University, Korea
Email: howon@korea.ac.kr

For a based CW-complex X, $\mathcal{A}^n_\sharp(X)$ is the submonoid of $[X, X]$ which consists of all homotopy classes of self-maps of X that induce an automorphism on $\pi_k(X)$ for all $0 \leq k \leq n$.

Since, for $m < n$, $\mathcal{A}^n_\sharp(X) \subseteq \mathcal{A}^m_\sharp(X)$, there is a chain by inclusions: $\mathcal{E}(X) \subseteq \mathcal{A}^\infty_\sharp(X) \subseteq ... \subseteq \mathcal{A}^1_\sharp(X) \subseteq \mathcal{A}^0_\sharp(X) = [X, X]$. We introduce the self closeness number and the self length. The self closeness number is the minimum number of $\mathcal{E}(X) = \mathcal{A}^k_\sharp(X)$. The self length is the number of strict inclusions in this chain for a given connected CW-complex. We has proved that the self closeness number is a homotopy invariant and prove self length is a homotopy invariant. We investigate the close connection with the self closeness number. Moreover, we determine self lengths of several spaces and provide the lower bounds or upper bounds of the self lengths of some spaces.

2010 MSC: Primary 55P10; Secondary 55Q05, 55Q55.

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Uniruleness of symplectic birational geometry on orbifolds

Bohui Chen, Chengyong Du and Jianxun Hu

Sichuan Normal University, China
Email: cyd9966@yahoo.com

Hu-Li-Ruan showed that the uniruleness is a symplectic birational invariant for smooth manifolds. Be precise, they defined uniruleness by testing the vanishingness of certain type of Gromov-Witten invariants and showed that such vanishing property is preserved by blow-ups and blow-downs. In this talk, we will explain that such results may be generalized to the orbifold case. We show that the similar vanishing property is preserved by any weighted blow-ups and blow-downs. This is based on the joint work with Bohui Chen and Jianxun Hu.

2010 MSC: Primary 53D45; Secondary 14N35.

String^c structures and modular invariants

Haibao Duan , Han Fei and Ruizhi Huang

Chinese Academy of Sciences, China
Email: huangrz@amss.ac.cn

Spin structure and its higher analogies play important roles in index theory and mathematical physics. In particular, Witten genera for String manifolds have nice geometric implications. As a generalization of the work of Chen-Han-Zhang (2011), we introduce the general Stringc structures based on the algebraic topology of Spinc groups. It turns out that there are infinitely many distinct universal Stringc structures indexed by the infinite cyclic group. Furthermore, we can also construct a family of the so-called generalized Witten genera for Spin^c manifolds, the geometric implications of which can be exploited in the presence of String^c structures. As in the un-twisted case studied by Witten, Liu, etc, in our context there are also integrality, modularity, and vanishing theorems for effective non-abelian group actions. We will also give some applications.

2010 MSC: Primary 58J26; Secondary 57S20, 53C27.

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Moduli spaces of Higgs bundles and hyper Kähler metrics

Zhenxi Huang

Sichuan University, China
Email:huangzhendong2011@163.com

A solution to the Yang-Mill's self-dual equation can be considered as an equivalent class of Higgs bundles. And the moduli spaces of Higgs bundles are the set of solutions to Yang-Mill's equations. These moduli spaces are complex symplectic manifolds with a natural hyper Kähler structures which can not be written explicitly in general. However, we can construct another hyper Kähler metric on the moduli spaces by thinking of them as integrable systems (known as Hitchin systems). The new Hyper Kähler metrics are called "semi-flat" metrics, since they are flat on the fibers of the integrable systems. In the talk, I will give results we have found last year in this topic.

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Upper bound for monoidal topological complexity

N. Iwase and M. Tsutaya

Kyushu Universiry, Japan
Email: iwase@math.kyushu-u.ac.jp

We show $\mathrm{tc}^{\mathrm{M}}(M) \leq 2\,\mathrm{cat}(M)$ for a finite simplicial complex M. For example, we have $\mathrm{tc}^{\mathrm{M}}(S^n \vee S^m) = 2$ for any positive integers n, m, which answers the question raised by Mark Grant.

2010 MSC: Primary 54H25, Secondary 55P50, 55T10.

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On nil groups for the quaternion group

Daniel Juan Pineda

UNAM, Mexico
Email: daniel@matmor.unam.mx

Nil groups are a large portion of the algebraic K theory groups of polynomial rings. We define these groups, some of its properties and an explicit example involving the quaternion group.

2010 MSC: Primary 19Axx; Secondary 18F30.

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Some applications of topological vertex algorithm

Xiaobin Li and Futoshi Yagi

Southwest Jiaotong University, China
Email: lixiaobin@home.swjtu.edu.cn

Topological vertex is an effective algorithm to compute Gromov-Witten invariants of toric Calabi-yau 3-fold for all degree and all genus. In this talk, I will introduce some of its applications to non-toric Calabi-Yau 3-folds and mirror symmetry.

2010 MSC: Primary 53D45; Secondary 14N35.

Representation theory of internal categories and its application to comodules over Hopf algebroids

Atsushi Yamaguchi

Osaka Prefecture University, Japan
Email: yamaguti@las.osakafu-u.ac.jp

J.F.Adams generalized the notion of Hopf algebras which are obtained from generalized homology theories satisfying certain conditions and showed that such a generalized homology theory takes values in the category of comodules over the "generalized Hopf algebra" associated with the generalized homology theory. This notion introduced by Adams is now called a Hopf algebroid which is a groupoid object in the opposite category of graded commutative rings. Under a "suitable framework", a comodule over a Hopf algebroid is a representation of the Hopf algebroid regarded as a groupoid.

The aim of this talk is to provide various fundamental notions on representations of internal categories (category object in a category with finite limits) by introducing a "suitable framework" making use of the notion of fibered category. Namely, we give definitions and constructions of restrictions, trivial representations, regular representations, left and right induced representations of internal categories. We also apply these gadgets to the fibered category of modules where the representations of Hopf algebroids are defined to see what happens to the category of comodules over a Hopf algebroid.

2010 MSC: Primary 18D30; Secondary 20L05.

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Converse of Smith theory

Min Yan and S. Cappell, S. Weinberger

Hong Kong University of Science and Technology, China
Email: mamyan@ust.hk

Suppose G is a finite group, and f is a map from a CW-complex F to the fixed point of a G-CW-complex Y. Is it possible to extend F to a finite G-CW complex X satisfying $X^G = F$, and extend f to a G-map $g\colon X \to Y$, such that g is a homotopy equivalence after forgetting the G-action?

In case Y is a single point, the problem becomes whether a given finite CW-complex F is the fixed point of a G-action on a finite contractible CW-complex. In 1942, P. A. Smith showed that the fixed point of a p-group action on a finite $\mathbb{Z}_p$-acyclic complex is still $\mathbb{Z}_p$-acyclic. In 1971, Lowell Jones proved a converse for semi-free cyclic group action on finite contractible X. In 1975, Robert Oliver proved that, for general action on finite contractible X, if the order of G is not prime power, then the only obstruction is the Euler characteristic of F.

We extend the classical results of Lowell Jones and Robert Oliver to the general setting. For semi-free action, we encounter a finiteness type obstruction. For general action by group of not prime power order, the obstruction is the Euler characteristics over components of Y^G. We calculate such obstructions for various examples.

2010 MSC: Primary 57Q91; Secondary 57Q12.

Free cyclic group actions on $(n-1)$-connected $2n$-manifolds

Jianqiang Yang

Honghe University, China
Email: yangjq_math@sina.com

In this paper we classify smooth orientation-preserving free actions of cyclic group $\mathbb{Z}/m$ on $\sharp g(S^n \times S^n)\sharp\Sigma$ and $K\sharp r(S^n \times S^n)$, where K is a $2n$-dimensional smooth Kervaire manifold. When $n = 2$ a classification up to topological conjugations is given. When $n = 3$ we obtain a complete classification up to smooth conjugations. For $n \geq 4$ a complete classification is given when the prime factors of m are large.

2010 MSC: Primary 53D45; Secondary 14N35.

Abstracts in General and Set-theoretic Topology

On the hyperspace of non-trivial convergent sequences

S. Garcia-Ferreira and Y. F. Ortiz-Castillo and R. Rojas-Herandez

UNAM, Mexico
Email: sgarcia@matmor.unam.mx

We consider the hyperspace of non-trivial convergent sequence equipped with the Vietories topology. We present some old and new results of these hyperspaces. Mainly, we consider connectedness and categorical properties.

2010 MSC: Primary 54B20, 54E52, 54F65; Secondary 54G12.

Usco selections and Choquet-completeness

Valentin Gutev

University of Malta, Malta
Email: valentin.gutev@um.edu.mt

The strong Choquet game is an infinite game between two players on a topological space selecting alternatively open sets by a rule. Spaces in which the second player has a winning strategy in any play of this game are called Choquet-complete. In the realm of metrizable spaces, Choquet-completeness is identical to complete metrizability. However, for general topological spaces it represents another interpretation of completeness. The present talk is about a recent result of the author that each regular space which admits an usco section for its Vietoris hyperspace of countable discrete sets is Choquet-complete. This result provides a natural generalisation of several known results for metrizable spaces, and offers other applications for non-metrizable spaces.

2010 MSC: 54B20, 54C60, 54E50, 91A44.

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Some recent progress on the study of gaps in the lattice of topological group topologies on a group

Wei He and Dekui Peng

Nanjing Normal University, P.R.China
Email: weihe@njnu.edu.cn

Let G be an abstract group and $\mathcal{S}$ be a subfamily of the lattice $\mathcal{G}(G)$ of all topological group topologies on G. A pair of elements $\tau, \sigma \in \mathcal{S}$ with $\sigma \subsetneq \tau$ is a *gap* in $\mathcal{S}$ if no element $\lambda \in \mathcal{S}$ satisfies $\sigma \subsetneq \lambda \subsetneq \tau$. In this talk, we will present some recent progress on the study of gaps in the lattice of topological group topologies on a group.

2010 MSC: Primary 22A05, 54A25; Secondary 54H11, 54A35

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Topological mixing and UPE

V. Martínez-de-la-Vega and D.Darji, A. Illanes, J. Martínez-Montejano

UNAM, México
Email: vmvm@matem.unam.mx

Abstract: In this talk we give relations and conditions for the maps mixing, weakly mixing and UPE to be equivalent on dendrites. This results allow us to generalize theorem of Darji & Kato that states the that if X is a G-like comtinuum for some graph G and G admits a 2-upe homeomorphism, then X is indecomposable. With this new results we can state weaker conditions and still obtain and indecomposable continuum.

2010 MSC: Primary 54H20; Secondary 54C70.

The inverse limit nonautonomous dynamical system

Manuel Sanchis and Gerardo Acosta

Universidad Jaume I, Spain
Email: sanchis@uji.es

In [1] the authors introduced the notion of the *inverse limit nonautonomous discrete dynamical system* (inverse limit NDS, for short) of an inverse sequence $(X_n, h_{\infty,n})_n$ of nonautonomous discrete dynamical systems, which generalizes the notions presented in [2, p. 53] of the inverse limit dynamical system and of the natural extension of an autonomous discrete dynamical system (X, f), using the shift map of f. In the present paper, the second dedicated by the authors to this topic, we continue the systematic study of the NDS inverse limit, by considering dynamical properties such as transitivity, point transitivity, mixing, strong transitivity, weakly mixing, ergodicity, denseness of periodic points, exactness and equicontinuity, among others. We generalize results obtained in [3], [4] and [5].

2010 MSC: 18A30, 37B55, 54H20, 54B99.

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A new class of topological groups containing all Polish groups and all minimal groups

Vitalij A. Chatyrko and Dmitri B. Shakhmatov

Ehime University, Japan
dmitri.shakhmatov@ehime-u.ac.jp

A bijective homomorphism between two groups is called an *isomorphism*. An isomorphism of a group G onto itself is called an *automorphism* of G.

Recall that a topological group G is *minimal* if every continuous isomorphism of G onto a Hausdorff group H is open. This class of topological groups is extensively studied in topological group theory; see the survey [2] and references therein.

We introduce a new class $\mathcal{R}$ of topological groups G having the following property: *every continuous automorphism of G is open*. Topological groups in the class $\mathcal{R}$ will be called *g-reversible*, an abbreviation for *group* reversible. This name was selected because the notion of a g-reversible group is a natural analogue of the notion of reversibility for topological spaces in the category of topological groups [1].

It is clear from the definitions that all minimal groups belong to the class $\mathcal{R}$.

According to a classical result of Banach, every continuous homomorphism of a Polish group onto another Polish group is open. Therefore, all Polish (=separable complete metric) groups belong to the class $\mathcal{R}$.

Furthermore, all locally compact σ-compact groups belong to the class $\mathcal{R}$, but not every locally compact abelian group belongs to $\mathcal{R}$.

Recall that a Hausdorff abelian group G is called *precompact* if it is a subgroup of some Hausdorff compact group, or equivalently, for each open neighbourhood U of zero of G there exists a finite subset F of G such that $U + F = G$. Every abelian group G has the strongest precompact group topology on G called its *Bohr topology*.

We show that every abelian group equipped with its Bohr topology belongs to the class $\mathcal{R}$, so every abelian group can be equipped with a g-reversible Hausdorff group topology. An example of a countable precompact metric abelian group outside the class $\mathcal{R}$ is constructed.

In the talk we shall survey our results from [1] related to the properties of members of the new class $\mathcal{R}$. A host of open problems will also be discussed.

2010 MSC: Primary 22A05; Secondary 20K30, 22B05, 22D05, 46A30, 54A10, 54D45, 54H11.

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Factoring continuous homomorphisms defined on subgroups of topological products

Mikhail Tkachenko

Universidad Autónoma Metropolitana, Mexico City, Mexico,
Email: mich@xanum.uam.mx

In 1944, Y. Mibu proved that every continuous real-valued function on an arbitrary product of compact spaces depends on at most countably many coordinates. This was a first step toward a long and very fruitful research in General Topology which can be called as *factoring continuous mappings on subspaces of product spaces.*

This line of research has its natural counterpart in Topological Algebra. In 1948, S. Kaplan shows as a byproduct that if $D = \prod_{i \in I} D_i$ is a product of topological abelian groups, then every continuous *character* of D depends on finitely many coordinates. As usual, a character of D is a homomorphism of D to the torus group $\mathbb{T}$. In fact, one can drop 'abelian' in the latter result.

In recent years, much attention has been paid to the study of different objects of Topological Algebra more general than topological groups. As for factoring continuous homomorphisms, Arhangel'skii and the author establish in 2008 that if S is an arbitrary subgroup of a product $D = \prod_{i \in I} D_i$ of *left topological groups* and $f : S \to K$ is a continuous homomorphism to a first countable left topological group K, then f depends on at most countably many coordinates. Furthermore, it turns out that one can find a countable subset J of the index set I and a *continuous* homomorphism $g : p_J(S) \to K$ satisfying $f = g \circ p_J \restriction S$, where p_J is the projection of D to $\prod_{i \in J} D_i$. We say that f has *countable type* in this case.

Our aim is to present several results on continuous homomorphisms defined on a *submonoid* S of a Cartesian product D of (semi)topological monoids. In a number of different situations, we show that every continuous homomorphism of S to a first countable topological group has countable type. If the codomain of the homomorphism is a Lie group, we can frequently show that the homomorphism has finite type. Typically, this happens if either the submonoid S fills all finite faces in the product D or S is *finitely retractable.*

All undefined terms will be explained in the lecture.

Epi-almost normality

Ibtesam Alshammari

University of Hafr Al Batin, Saudi Arabia
Email: iealshamri@uhb.edu.sa

A space $(X , \mathcal{T})$ is called *epi-almost normal* if there exists a coarser topology $\mathcal{T}'$ on X such that $(X , \mathcal{T}')$ is Hausdorff (T_2) almost normal. We investigate this property and present some examples to illustrate the relationships between epi-almost normality and other weaker kinds of normality.

2010 MSC: 54D15, 54D10.

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A useful common weakening of Lindelöf and H-closed

Angelo Bella

University of Catania, Italy
Email: bella@dmi.unict.it

The inequality $|X| \leq 2^{\chi(X)}$ has been proved to be true for both the class of Lindelöf spaces (Arhangel'skii, 1969) and that of H-closed spaces (Dow-Porter, 1982), by different arguments. We present a common weakening of the Lindelöf and H-closed properties which allows us to give a unified proof of these two theorems.

2010 MSC: Primary 54A25, 54D20, 54D55.

Some characterizations of Mildly $\mathcal{I}$-Hurewicz covering property

Manoj Bhardwaj and B. K. Tyagi

University of Delhi, India
Email: manojmnj27@gmail.com

A space X is said to have *$M\mathcal{I}$-Hurewicz property* (in short $M\mathcal{I}H$) (Das.,2018) if for each sequence $< \mathcal{U}_n : n \in \omega >$ of clopen covers of X there is a sequence $< \mathcal{V}_n : n \in \omega >$ such that for each n, $\mathcal{V}_n$ is a finite subset of $\mathcal{U}_n$ and for each $x \in X$, $\{n \in \omega : x \notin \bigcup \mathcal{V}_n\} \in \mathcal{I}$. In this paper, we continue to investigate topological properties of $M\mathcal{I}H$. We characterized $M\mathcal{I}H$ property by $M\mathcal{I}$-Hurewicz Basis property and $M\mathcal{I}$-Hurewicz measure zero property for metrizable spaces. We also characterized mildly star-$\mathcal{I}$-Hurewicz property using selection principles.

2010 MSC: Primary 54D20; Secondary 54B20.

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A note on paratopological groups with an ω^ω-base

Zhangyong Cai, Peiqi Ye, Shou Lin and Bin Zhao

Nanning Normal University, P.R. China
Email: zycaigxu2002@126.com

In this paper, paratopological groups with an ω^ω-base are investigated. The following results are obtained, which generalizes some conclusions in literature. (1) Every Fréchet-Urysohn Hausdorff paratopological group having the property (**) with an ω^ω-base is first-countable, hence submetrizable, where a paratopological group G has the property (**) if there exist a non-trivial sequence $\{x_n\}_{n\in\mathbb{N}}$ in G such that both $\{x_n\}_{n\in\mathbb{N}}$ and $\{x_n^{-1}\}_{n\in\mathbb{N}}$ converge to the identity of G. (2) The free Abelian paratopological group $AP(X)$ on a topological space X has an ω^ω-base if and only if the fine quasi-uniformity of X has an ω^ω-base. (3) If X is a countable topological space, then the free paratopological group $FP(X)$ on X has an ω^ω-base if and only if the fine quasi-uniformity of X has an ω^ω-base.

2010 MSC: Primary 22A30; Secondary 54D70.

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On quasi-continuous dynamical systems

Jiling Cao and Aisling McCluskey

Auckland University of Technology, New Zealand
Email: jiling.cao@aut.ac.nz

The concept of a quasi-continuous function can be traced back to Volterra, see [1,p.95]. However, a formal definition was introduced by Kempisty [3] for a real-valued function, as an important generalization of continuous functions. It is well known that under appropriate conditions on its domain and co-domain, a quasi-continuous function is continuous on a residual set of its domain. Due to the fact that many standard examples from dynamical systems, including the baker transformation and interval exchange maps, are quasi-continuous, Crannell and Martelli [2] studied quasi-continuous dynamical systems on compact metric spaces in 2000. In this paper, we study quasi-continuous dynamical systems on general topological spaces. We extend some standard results of continuous dynamical systems on metric spaces, such as transitivity and dense periodic points, etc,. to quasi-continuous dynamical systems on general topological spaces.

2010 MSC: Primary 37B99; Secondary 54H20.

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Topological groups with a property of Dierolf and Warken

Dmitri Shakhmatov and Víctor Hugo Yañez

Ehime University, Japan.
victor_yanez@comunidad.unam.mx

A topological group is *minimally almost periodic* (MinAP) if every continuous homomorphism of it to a compact group is trivial. The theory of minimally almost periodic groups has deep connections with harmonic and functional analysis, as well as topological dynamics. Dierolf and Warken contributed one of the fundamental results to this theory: every topological group can be embedded as a closed subgroup of some MinAP topological group [2]. They proved this result by showing that the Hartman-Mycielski [6] construction of any topological group carries some new property which implies MinAP. This new property was later noticed by Gould [1, 5], who introduced and studied a weaker version of it which still implies MinAP. Gould called his property the *small subgroup generating property* (SSGP).

Dikranjan and Shakhmatov has given a complete characterization of abelian groups which admit a MinAP group topology [4]. They also gave an "almost complete" characterization of abelian groups admitting an SSGP group topology [3]. The remaining case was settled by the present authors in [7].

In this talk, we shall re-examine the *original* property of Dierolf and Warken which we call the *algebraic* small subgroup generating property (ASSGP). The reasoning behind this name will become clear from the definition below.

For a subset A of a group G, we denote by $\langle A\rangle$ the smallest subgroup of G containing A and let $\mathrm{Cyc}(A) = \{x \in G : \langle x\rangle \subseteq A\}$.

Definition 1. Let G be a topological group. We say that

(i) G has the small subgroup generating property (SSGP) if the set $\langle \mathrm{Cyc}(U)\rangle$ is dense in G for every neighbourhood U of the identity of G;

(ii) G has the *algebraic* small subgroup generating property (ASSGP) if $\langle \mathrm{Cyc}(U)\rangle = G$ holds for every neighbourhood U of the identity of G.

These properties are related by the following diagram of (non-reversible) implications:

$$\text{ASSGP} \to \text{SSGP} \to \text{MinAP}. \tag{1}$$

The goal of this talk is to present new results achieved [8, 9, 10] for groups belonging to the class of ASSGP groups. We shall also highlight the striking difference between the groups in this class and the groups in the SSGP class of Gould.

2010 MSC: Primary 22A05; Secondary 20K15, 20K21, 20K25, 20K27, 54H11.

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Semi-Kelley continua

Alejandro Illanes

UNAM, México
Email: illanes@matem.unam.mx

Let X be a continuum (compact connected metric space). Some useful concepts to study continua are Kelley property and semi-Kelley property. In this talk se will discuss how the property of being semi-Kelley behaves with respect to products, cones, suspensions, contractibility of hiperspaces, smoothness, and mappings preserving the property of being semi-Kelley.

2010 MSC: Primary 54F50; Secondary 54B20.

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Point-regular covers and sequence-covering compact mappings

Shou Lin, Xuewei Ling and Ying Ge

Ningde Normal University, P.R. China
Email: shoulin60@163.com

In this paper, we discuss some relations among point-regular covers, uniform covers and σ-point-finite covers of a topological space, express spaces with a point-regular special family as images of metric spaces under certain compact mappings, and prove that the following statements are equivalent for a subset A of a topological space X.

(1) X has a point-regular cs-network at A for X.
(2) X has a point-regular sn-network at A for X.
(3) X has a uniform cs-network at A for X.

(4) X has a uniform sn-network at A for X.

(5) There are a metric space M and a mapping $f : M \to X$ satisfying the following conditions: for each $x \in A$, $f^{-1}(x)$ is compact in M; and there is a point $z \in f^{-1}(x)$ such that $f(U)$ is a sequential neighborhood of x in X whenever U is a neighborhood of z in M.
As some applications of these results, some characterizations of certain compact images of metric spaces are obtained.

2010 MSC: Primary 54E40; Secondary 54C10; 54D55; 54D70; 54E35; 54E99.

The hyperspace of ω limit sets

Jorge M. Martínez-Montejano and Yajaida N. Velázquez-Inzunza

UNAM, México
Email: jorgemm@ciencias.unam.mx

Given a dynamical system (X, f) with X compact metric, for each $p \in X$, we define $\omega(p, f)$ as the set of limit points of the orbit of p under f; and $\omega(f) = \{\omega(x, f) : x \in X\}$. We consider $\omega(f)$ as a subspace of the hyperspace of the closed and nonempty subsets of X with the Hausdorff metric. We show that for a class of dendrites $\mathcal{D}$ (that contains all the dendrites with closed set of end points) the following is true. If $D \in \mathcal{D}$ and $f : D \to D$ is a transitive map, then $\omega(f)$ is totally disconnected. Also, we give an example of a transitive map f on the universal dendrite such that $\omega(f)$ is not totally disconnected.

2010 MSC: Primary 54H20; Secondary 54B20.

On the diagonals of topological spaces

Lei Mou, Piyu Li, Yang Gao and Shou Lin

Capital Normal University, Beijing, China
Email: moulei@cnu.edu.cn

In [1], Arhangel'skii and Buzyakova gave the following conjecture: For each $m \in \omega$, there is a Tychonoff space X_m with a rank m-diagonal that is not a rank $m{+}1$-diagonal.

In [2], the authors constructed a Tychonoff Moore space that has a diagonal of the rank exactly 4 and a zero-set diagonal. In [10], Yu and Yun proved that for each $m \geq 4$, there exists a separable Tychonoff Moore space with a diagonal of the rank exactly m. In [3], we proved that for each $m \geq 5$, there exists a Tychonoff Moore space with a diagonal of the rank exactly m and a zero-set diagonal. In [3], we also constructed a non-submetrizable Tychonoff Moore space with a diagonal of infinite rank and a zero-set diagonal. Note that each separable space with a zero-set diagonal is submetrizable [6].

Let $\mathcal{A}$ be an almost disjoint family of infinite subsets of $\mathbb{N}$. The Mrówka space $\Psi(\mathcal{A})$ denotes the space with underlying set $\mathbb{N} \cup \mathcal{A}$ and with the topology having as a base all singletons $\{n\}$

for $n \in \mathbb{N}$ and all sets of the form $\{A\} \cup (A \setminus F)$, where $A \in \mathcal{A}$ and F is finite. Mrówka [9] proved that $\Psi(\mathcal{A})$ is pseudocompact if and only if $\mathcal{A}$ is a MADF (i.e., maximal almost disjoint family). McArthur [7] proved that every pseudocompact space with a regular G_δ-diagonal is metrizable. Hence, $\Psi(\mathcal{A})$ has no regular G_δ-diagonal if $\mathcal{A}$ is a MADF. If $\mathcal{A}$ be a MADF and $\mathcal{B}$ is a countable subsetof $\mathcal{A}$, then Mrówka space $\Psi(\mathcal{A} \setminus \mathcal{B})$ has no regular G_δ-diagonal [5]. If $\mathcal{A}$ is a MADF and $\mathcal{B} \subseteq \mathcal{A}$ is of size $< \mathfrak{a}$, then the Mrówka space $\Psi(\mathcal{A} \setminus \mathcal{B})$ has no regular G_δ-diagonal [8], where $\mathfrak{a} = \min\{|\mathcal{A}| : \mathcal{A}$ is an MADF on $\mathbb{N}\}$.

In [5], the following question is asked: Let $\mathcal{A}$ be an almost disjoint family of infinite subsets of $\mathbb{N}$. If $\Psi(\mathcal{A})$ has a regular G_δ-diagonal, Is $\Psi(\mathcal{A})$ is submetrizable? In [4], we prove that for each natural number $m \geq 3$, there exists a Mrówka space with a diagonal of the rank exactly m, which give a negative answer to this question.

We also discuss under what kind of maps the property of having a regular G_δ-diagonal (or zero-set diagonal) is preserved or inversely preserved.

2010 MSC: Primary 54C10; 54E30; 54E99.

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Means on continua

Jimmy A. Naranjo-Murillo

UNAM, México
Email: jimmy.naranjo@matem.unam.mx

A *mean* for a continuum X is a continuous function $m : X \times X \to X$ such that $m(x, x) = x$ and $m(x, y) = m(y, x)$ for every $x, y \in X$. It is known that continua admitting means must be unicoherent [1] and continua of type N do not admit means [2, Corollary 2.3, p. 266]. A problem that remains unsolved is whether continua of generalized type N can admit means [3, Question 3.25, p. 64].

In this talk we will discuss what is known about means on continua. We will also present new results concerning means on dendroids of generalized type N with a simple triod as limit and some open questions.

2010 MSC: Primary 54B20; Secondary 54F50.

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On some properties of PIGO and PIGT

Liang-Xue Peng and Huan Wang

Beijing University of Technology, China
Email: pengliangxue@bjut.edu.cn

Let PIGO be the class of perfect images of generalized ordered (GO) spaces. We give some equivalent conditions that an element of PIGO is a D-space.

Let PIGT be the class of perfect images of generalized trees with Hausdorff generalized Sorgenfrey topologies. If $X \in PIGT$ and X is a k-semistratifiable (stratifiable) space, then X is metrizable. If $X \in PIGT$, then X is first-countable if and only if X has a countable pseudocharacter. If $f : L \to X$ is a continuous irreducible mapping from the GO-space L (the generalized tree L with a Hausdorff generalized Sorgenfrey topology) onto X, then X has a regular G_δ-diagonal (G_δ^*-diagonal), so does L.

2010 MSC: Primary 54D35; Secondary 54C10, 54E20.

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Lindelöfness and Cech-completeness on the hypergroup of Hartman-Mycielski

Iván Sánchez
Email: isr.uami@gmail.com

Given an arbitrary topological group G, then G can be embedded as a closed subgroup of a pathwise connected and locally pathwise connected topological group $G^\bullet$. This result was proved by Hartman and Mycielski in 1958. By this reason, we say that $G^\bullet$ is *the hypergroup of Hartman-Mycielski associated* to G.

It is known that G and $G^\bullet$ share some properties: metrizability, second countability, separability, σ-compactness, among others. In this talk, we characterize when $G^\bullet$ is Lindelöf. We also say when $G^\bullet$ is Cech-complete. These results answer some questions posed by Arhangel'skii and Tkachenko.

On countably selective spaces

Ziqin Feng, Gary Gruenhage and Rongxin Shen

Taizhou University, China
Email: srx20212021@163.com

A space X is strongly Y-selective (resp., Y-selective) if every lower semicontinuous mapping from Y to the nonempty subsets (resp., nonempty closed subsets) of X has a continuous selection. We also call X (strongly) C-selective if it is (strongly) Y-selective for any countable and regular space Y. E. Michael showed that every first countable space is strongly C-selective. We extend this by showing that every W-space in the sense of the second author is strongly C-selective. We also show that every GO-space is C-selective, and that every $(\omega+1)$-selective space has Arhangel'skii's property α_1. We obtain an example under $\mathfrak{p}=\mathfrak{c}$ of a strongly $(\omega+1)$-selective space that is not C-selective, and we show that it is consistent with and independent of ZFC that a space is strongly $(\omega+1)$-selective iff it is $(\omega+1)$-selective and Fréchet. Finally, we answer a question of the third author and Junnila by showing that the ordinal space ω_1+1 is not self-selective.

2010 MSC: Primary 54C65.

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Uniform topology on countable subsets

Ángel Tamariz-Mascarúa A. D. Rojas, R. Pichardo y H. Villegas

UNAM, México
Email: atamariz@unam.mx

In this talk we analyze topologies $\mathcal{T}$ on the set $C(X)$ of real-valued continuous functions defined on X which satisfy: if a sequence $(f_n)_{n<\omega}$ in $C(X)$ converges to f in $(C(X), \mathcal{T})$, then $(f_n)_{n<\omega}$ uniformly converges to f. In particular, we study the topology $\mathcal{S}$ of the uniform convergence on the countable subsets of X. We present some results on compactness and completeness type properties and on cardinal functions concerning $(C(X), \mathcal{S})$.

A study on D-spaces and the metrization of compact spaces with property (σ-A)

Huan Wang and Liangxue Peng

Beijing University of Technology, China
Email: wanghuan2015@emails.bjut.edu.cn

In this article, we introduce two notions which are called property (σ-A) and property (σ-B). They are generalizations of property (A) and property (B)[1], respectively. We show that every compact Hausdorff space which satisfies property (σ-A) is metrizable. Every space with a point-countable base satisfies strong property (σ-A). Every space which has Collins-Roscoe property satisfies property (σ-B).

We show that the properties of property (σ-A) and property (σ-B) are closed under finite products. A finite products of T_1-spaces which satisfy property (σ-B) (property (σ)-A, σ-sheltering (F), σ-well-ordered (F)) is hereditarily a D-space. If (X, $\mathcal{T}$) satisfies ω_1-sheltering (F), then (X, $\mathcal{T}_\omega$) is hereditarily a D-space. We finally show that if a space X satisfies ω_1-sheltering (F) and every countable discrete subspace of X is closed, then X is hereditarily a D-space. This gives a partial answer to a question posed by Z.Q. Feng and J.E. Porter in 2015[2].

2010 MSC: Primary 54D20; Secondary 54F99.

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The progresses of PT-groups and Dieudonné completion of paratopological groups

Li-Hong Xie

Wuyi University, China
Email: xielihong2011@aliyun.com

In 1985 V.G. Pestov and M.G. Tkachenko asked the next question:

Problem: Let G be a topological group, and μG the Dieudonné completion of the space G. Can the operations in G be extended to μG in such a way that μG becomes a topological group containing G as a topological subgroup?

In 2000, Arhangel'skii gives a negative answer to this problem. Also, he called a topological group G a PT-*group*, if the operations on G can be extended to the Dieudonné completion μG in such a way that G becomes a topological subgroup of μG. In this talk, we shall give a survey on PT-groups and Dieudonné completion of paratopological groups.

2010 MSC: Primary 54D50, 54D60; Secondary 54C35.

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Three results independent of ZFC for certain products

Yukinobu Yajima and Yasushi Hirata

Kanagawa University, Japan
Email: yajimy01@kanagawa-u.ac.jp

Let $\mathbb{N}$ be the discrete space of natural numbers. For a cardinal $\kappa > \omega$, let $\mathbb{N}^\kappa$ be the product of κ many copies of $\mathbb{N}$.

Recall $\mathfrak{b} = \min\{|\mathcal{B}| : \mathcal{B} \subset {}^\omega\omega$ is an unbounded family$\}$.

Theorem 1 ([1]). Assuming $\mathfrak{b} > \omega_1$, every C^*-embedded subset in $\mathbb{N}^{\omega_1}$ is C-embedded in $\mathbb{N}^{\omega_1}$.

We denote by CH the continuum hypothesis.

Theorem 2 ([2]). Assuming CH, every C^*-embedded discrete subset in $\mathbb{N}^{\mathfrak{c}}$ is countable.

Let $e(X) = \omega \cdot \sup\{|D| : D$ is closed discrete subset in $X\}$, called the *extent* of a space X.
A cardinal κ is *weakly inaccessible* if it is regular limit cardinal $> \omega$.

Theorem 3. Let A and B be subspaces of an ordinal. Then $A \times B$ is countably paracompact if and only if $e(A' \times B') = e(A') \cdot e(B')$ for each closed rectangle $A' \times B'$ in $A \times B$, under the non-existence of a weakly inaccessible cardinal.

The three results above are not only consistent but also independent of ZFC.

At the end, I would like to introduce a preview of the animation "Linear and Algebra and the World" with English subtitles, which consists of the three parts.
These have been uploaded in "https://www.youtube.com/user/kanagawaunivofficial", and all other ones are listed in "http://www.math.kanagawa-u.ac.jp/contentsVideo.html".

2010 MSC: Primary 03E10, 54B10, 54C45; Secondary 03E35, 54A25.

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Topological properties of function space with hypo-graph topology

Hanbiao Yang

Wuyi university, China
Email:hongsejulebu@sina.com

The main research of the talk is the topological properties of function space with hypo-graph topology. $C(X,T)$ is the function space, from a non-degenerate Peano continuum to a tree with n segments $S_1, S_2, \ldots, S_n$. The concrete research results are $C(X,T) \approx \oplus CUB(S_i)$, where $CUB(S_i) = \{f$ is in $C(X,T) : \max f(X)$ is in $S_i\}$ and $CUB(S_i)$ is an AR.

2010 MSC: Primary 54B20; Secondary 54C35.

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On retractional skeletons on compact spaces

Cenobio Yescas-Aparicio and Salvador García-Ferreira

UNAM, México
Email:novo1126@hotmail.com

It is know that the Valdivia compact spaces can be characterized by a family of retractions called retractional skeleton (see [1]). Also we know that there are compact spaces with retractional skeletons which are not Valdivia. The standard definition of a Valdivia compact spaces is via a Σ-product of a power of the unit interval. Following this fact we introduce the notion of π-skeleton on a compact space X by embedding X in a suitable power of the unit interval together with a pair $(\mathcal{F}, \varphi)$, where $\mathcal{F}$ is family of cosmic subspaces of X and φ an ω-monotone function which satisfy certain properties. This new notion generalize the idea of a Σ-product. We prove that a compact space admits a retractional skeleton iff it admits a π-skeleton. This equivalence allows to give a new proof of the fact that the product of compact spaces with retractional skeletons admits an retractional skeleton (see [2]).

2010 MSC: Primary 54D30, 54C15.

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Abstracts in Geometric Topology

Simultaneous extension of equivariant maps

Sergey A. Antonyan and Lili Zhang

UNAM, Mexico
Email: antonyan@unam.mx

Let X be a metrizable space, A a closed subset of X, and L a locally convex topological vector space. Let $C(X,L)$ denote the vector space of continuous functions from X into L, and similarly for $C(A,L)$. We equip these function spaces with the compact-open topology. The famuous Dugundji extension theorem asserts that for every $f \in C(A,L)$ there exists $\Lambda(f) \in C(X,L$ such that $\Lambda(f)|_A = f$ and $\mathcal{I}m\,\Lambda(f) \subset conv\,(\mathcal{I}m\,f)$. In 1953, E. Michael and R. Arens independently observed that the map $\Lambda : C(A,L) \to C(Z,L)$ constructed by Dugundji is, in fact, a linear homeomorphic embedding. In this talk we will discuss how to extend this result to the category of G-spaces. We will prove that an analogous equivariant result is true when the acting group G is compact Lie. By an example, we will show that the result may fail to be true when G is a compact non-Lie group.

The speaker thanks the Department of Mathematics of the Xi'an Technological University, where this joint research with Lili Zhang was completed.

Upper and lower topologies on hyperspaces

Natalia Jonard-Pérez

UNAM, Mexico
Email: nat@ciencias.unam.mx

Let X be a topological space and $CL(X)$ the family of all closed subsets of X. Classical topologies on the hyperspace $CL(X)$ are the symmetrization (supremum) of two weaker topologies. For example, the Vietoris topology is the minimum topology containing the *lower Vietoris topology* (the topology generated by the sets of the form $U^- := \{A \in CL(X) \mid A \cap U \neq \emptyset\}$ where $U \subset X$ is open) and the *upper Vietoris topology* (the one generated by the sets of the form $W^+ := \{A \in CL(X) \mid A \subset W\}$, where W is an open subset of X). A similar situation occurs with the Fell topology (or any other "hit and miss topology") and the Hausdorff metric topology.

In this talk we will see that each of these weaker topologies is responsable for different topological properties of the hyperspace $CL(X)$. In particular, we will show that if X is a G-space, the continuity of the action of G on the hyperspace (with respect to one of these weaker topolopogies) characterizes certain topological properties of the group G.

This is a joint work with Víctor Donjuán.

Characterizing infinite-dimensional manifolds by general position properties

Katsuhisa Koshino

Kanagawa University, Japan
Email: ft160229no@kanagawa-u.ac.jp

In this talk, spaces are metrizable, maps are continuous, and $\aleph_0 \leq \kappa < \lambda$. Let $\ell_2^f(\kappa)$ be the linear span of the canonical orthonormal basis in the Hilbert space $\ell_2(\kappa)$ of density κ. A closed set A is a *Z-set* in a space X if for each open cover $\mathcal{U}$ of X, there exists $f : X \to X$ such that f is $\mathcal{U}$-close to the identity map on X and $f(X) \cap A = \emptyset$. When the closure of $f(X)$ misses A, it is called a *strong Z-set*. A *Z-embedding* is an embedding whose image is a Z-set. Given a class $\mathfrak{C}$, we say that X is *strongly $\mathfrak{C}$-universal* if the following condition is satisfied.

- Let $A \in \mathfrak{C}$ and $f : A \to X$ be a map. Suppose that B is a closed subset of A and the restriction $f|_B$ is a Z-embedding. Then for every open cover $\mathcal{U}$ of X, there exists a Z-embedding $g : A \to X$ such that g is $\mathcal{U}$-close to f and $g|_B = f|_B$.

For spaces $X \subset M$, X is called to be *homotopy dense* in M if there is a homotopy $h : M \times \mathbf{I} \to M$ such that $h(M \times (0, 1]) \subset X$ and $h(x, 0) = x$ for any $x \in M$. A *strong Z_σ-set* is a countable union of strong Z-sets. It is said that X is a *$\mathfrak{C}$-absorbing set* in M provided that $X \in \mathfrak{C}_\sigma$, is homotopy dense in M, is a strong Z_σ-set in itself, and is strongly $\mathfrak{C}$-universal, where $\mathfrak{C}_\sigma = \{\bigcup_{n<\aleph_0} A_n \mid A_n \in \mathfrak{C}$ and A_n is closed$\}$.

A space X has *the κ-discrete approximation property* for $\mathfrak{C}$, which is a general position property based on "density" of infinite-dimensional manifolds, if the following condition holds.

- Let $f : \bigoplus_{\gamma<\kappa} A_\gamma \to X$ be a map from a topological sum of spaces in $\mathfrak{C}$. For every open cover $\mathcal{U}$ of X, there is $g : \bigoplus_{\gamma<\kappa} A_\gamma \to X$ such that g is $\mathcal{U}$-close to f and $\{g(A_\gamma) \mid \gamma < \kappa\}$ is discrete in X.

In the case that $\mathfrak{C} = \{\mathbf{I}^n\}$, $n < \aleph_0$, X has *the κ-discrete n-cells property.* We say that $\mathfrak{C}$ is *topological* if any space homeomorphic to some member of $\mathfrak{C}$ also belongs to $\mathfrak{C}$, and is *closed hereditary* if any closed subspace of some member of $\mathfrak{C}$ also belongs to $\mathfrak{C}$. By the discrete cells property, topological manifolds modeled on absorbing sets in Hilbert spaces can be characterized as follows (cf. [1, 3, 2]).

Theorem. *Let $\mathfrak{C}$ be a topological and closed hereditary class, and Ω be a $\mathfrak{C}$-absorbing set in $\ell_2(\kappa)$. For a connected space $X \in \mathfrak{C}_\sigma$ of density $\leq \kappa$, X is an Ω-manifold if and only if the following conditions hold.*

1. *X is an ANR and a strong Z_σ-set in itself.*
2. *X is strongly $\mathfrak{C}$-universal.*
3. *X has the κ-discrete n-cells property for every $n < \aleph_0$.*

A class $\mathfrak{C}$ is called to be $\mathbf{I}$-*stable* if $A \times \mathbf{I} \in \mathfrak{C}$ for each $A \in \mathfrak{C}$. For a topological, closed hereditary and $\mathbf{I}$-stable class $\mathfrak{C}$, if Ω is a $\mathfrak{C}$-absorbing set in $\ell_2(\kappa)$, then $\ell_2^f(\lambda) \times \Omega$ is a $\bigoplus_\lambda \mathfrak{C}(\kappa)$-absorbing set in $\ell_2(\lambda) \times \ell_2(\kappa)$. Here $\bigoplus_\kappa \mathfrak{C} = \{\bigoplus_{\gamma<\kappa} A_\gamma \mid A_\gamma \in \mathfrak{C}\}$ and $\mathfrak{C}(\kappa) = \{A \in \mathfrak{C} \mid A$ is of density $\leq \kappa\}$. Using the discrete cells property, we can establish the following characterization.

Main Theorem. *Suppose that $\mathfrak{C}$ is a topological, closed hereditary and $\mathbf{I}$-stable class, and that Ω is a $\mathfrak{C}$-absorbing set in $\ell_2(\kappa)$. A connected space $X \in (\bigoplus_\lambda \mathfrak{C}(\kappa))_\sigma$ is an $(\ell_2^f(\lambda) \times \Omega)$-manifold if and only if the following conditions are satisfied.*

1. *X is an ANR and a strong Z_σ-set in itself.*
2. *X is strongly $\mathfrak{C}(\kappa)$-universal.*
3. *X has the λ-discrete n-cells property for any $n < \aleph_0$.*

2010 MSC: Primary 57N20; Secondary 54F65, 57N75.

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Gromov-Witten invariants and virtual neighborhoods

Li Sheng

Sichuan University, China
Email: lsheng@scu.edu.cn

We use the approach of Ruan and Li-Ruan to construct virtual neighborhoods and show that the Gromov-Witten invariants can be defined as an integral over top strata of virtual neighborhood. The invariants defined in this way satisfy all the Gromov-Witten axioms of Kontsevich and Manin.

D-branes, K-homology and index theory

Bai-Ling Wang

The Australian National University, Australia
Email: bai-ling.wang@anu.edu.au

K-homology is the dual to K-theory. For a locally finite CW-complex X, there are three equivalent ways to define its K-homology: homotopy theory, functional analysis and geometric cycles. In string theory, D-branes in a spacetime are geometric objects where open strings can end with Dirichlet boundary conditions. In this talk, I will describe these geometric cycles for any topological (orbifold) CW complex and their applications to index theory.

2010 MSC: Primary 19L50; Secondary 19K35.

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Some geometric correspondences for homothetic navigations

Ming Xu, Vladimir Matveev, Ke Yan and Shaoxiang Zhang

Capital Normal University, China
Email: mgmgmgxu@163.com

Zermero navigation is important in Finsler geometry for constructing new metrics and studying their geometry. If we use homothetic vector field in the navigation, many geometry features, like geodesics and orthogonal Jacobi

fields before and after the navigation can be naturally corresponded. Using these natural and simple observations, we found an easy proof for X. Mo and L. Huang's flag curvature formula, which is also valid in pseudo-Finsler context. Furthermore, we compare the Busemann-Hausedorff volumn forms and the Laplacian of a transnormal function before and after a homothetic navigation, then we directly get a correspondence for the isoparametric hypersurfaces. This result generalized the works of Q. He and her coworkers, on the classification of isoparametric hypersurfaces in Funk spaces and Randers space forms by studying their submanifold geometry.

2010 MSC: Primary 53C60; Secondary 53C42.

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The mixed Pólya-Szegö principle and its applications

Niufa Fang and Jiazu Zhou

Chern Institute of Mathematics, Nankai University, China
Email:fangniufa@nankai.edu.cn

The classical Pólya-Szegö principle plays a fundamental role in the solution to a number of variational problems in different areas such as isoperimetric inequalities, Sobolev inequalities, etc. It states that the L_p norm of the gradient of a function in $\mathbb{R}^n$ (with L_p partial dervative) does not increase under symmetric rearrangement.

In this talk, we will introduce the mixed Pólya-Szegö principle. When $1 \leq p < n$, the mixed Pólya-Szegö principle implies a special L_p Minkowski inequality; when $p = n$, the mixed Pólya-Szegö principle deduces the mixed Morrey-Sobolev; when $p > n$, the mixed Moser-Trudinger inequality follows from the mixed Pólya-Szegö principle. This talk includes some joint works with professor Jiazu Zhou.

2010 MSC: Primary 52A40; Secondary 52A20.

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Isoperimetric inequalities for (p, q)-mixed geominimal surface and (p, q)-mixed affine surface area

Xiao Li, H. Wang and J. Zhou
Email: lxlixiaolx@163.com

Lutwak, Yang and Zhang investigated L_p dual curvature measure. Motivated by works of Lutwak, Yang and Zhang [1], (p, q)-mixed geominimal surface area and (p, q)-mixed affine surface area are introduced. These are natural extensions of L_p geominimal surface area and L_p affine surface area. Associated isoperimetric inequalities for (p, q)-mixed geominimal surface area and (p, q)-mixed affine surface area are obtained.

2010 MSC: Primary 52A20; Secondary 53A15.

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[2] X. Li, H. Wang and J. Zhou, (p, q)-mixed geominimal surface area and (p, q)-mixed affine surface area, *J. Math. Anal. Appl.* **475** (2019), 1472-1492.

A Kähler structure of the PU$(2, 1)$ configuration space of four points in S^3

Li-Jie Sun

From the strictly pseudoconvex CR structure of the boundary of the Siegel domain model for complex hyperbolic plane $\mathbf{H}^2_{\mathbb{C}}$ (which is isomorphic to the Heisenberg group $\mathfrak{H}$), we construct a Kähler structure of the Riemannian cone $\mathfrak{H}^\star \times \mathbb{R}_{>0}$, where $\mathfrak{H}^\star$ is the hyperbolic Heisenberg group.

In this talk we mainly prove that the PU$(2, 1)$ configuration space $\mathfrak{F}_4$ of four points in S^3 is in bijection with the Kähler cone of hyperbolic Heisenberg group. Time permitting, we will also show the relations of the well-known structures of $\mathfrak{F}_4$ with the new structure.

This is a joint work with Ioannis D. Platis.

Dispersionless integrable hierarchy via Kodaira-Spencer gravity

Xinxing Tang and Weiqiang He, Si Li, Philsang Yoo

YMSC, Tsinghua University, China
Email: tangxinxing@mail.tsinghua.edu.cn

In this talk, I will explain how dispersionless integrable hierarchy in 2d topological field theory arises from the Kodaira-Spencer gravity (BCOV theory [1],[2]). The infinitely many

commuting Hamiltonians are given by the current observables associated to the infinite abelian symmetries of the Kodaira-Spencer gravity. A BV framework of effective field theories will be introduced that leads to the B-model interpretation of dispersionless integrable hierarchy.

2010 MSC: Primary 51P05; Secondary 37K05.

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Rational cubic fourfolds in Hassett divisors

Song Yang

Tianjin University, China
Email: syangmath@tju.edu.cn

I will give a brief review on the rationality problem of smooth cubic fourfolds, and present that every Hassett's Noether-Lefschetz divisor contains a union of three codimension-two subvarieties, parametrizing rational cubic fourfolds, in the moduli space of smooth cubic fourfolds. This is based on a joint work with Xun Yu.

Dolbeault cohomologies of blowing up complex manifolds

Xiangdong Yang, Sheng Rao and Song Yang

Chongqing University, China
Email: math.yang@cqu.edu.cn

For a compact complex manifold, the Dolbeault double complex gives rise to a basic cohomological invariant: Dolbeault cohomology. From bimeromorphic geometry point of view, it is fundamental to study the variation problem of this invariant under blow-ups. In this talk, we explain a sheaf-theoretic approach to prove a blow-up formula for Dolbeault cohomologies of compact complex manifolds by introducing relative Dolbeault cohomology. As an application, we obtain the bimeromorphic stability for degeneracy of the Frölicher spectral sequences at E_1 on compact complex threefolds and fourfolds.

2010 MSC: Primary 14E05; Secondary 18G40.

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Dual Orlicz-Brunn-Minkowski theory

Baocheng Zhu, Deping Ye and Jiazu Zhou

Hubei Minzu University, China
Email: zhubaocheng814@163.com

The dual Brunn-Minkowski theory, initiated by Lutwak, provides powerful tools to solve the long-standing Busemann-Petty problem in the 1990's. Among those deep results, the dual Brunn-Minkowski and dual Minkowski inequalities are the most important.

In this talk, I will discuss the newly introduced dual Orlicz-Brunn-Minkowski theory, a nontrivial extension of the dual Brunn-Minkowski theory. In particular, I will talk about the dual Orlicz-Minkowski and dual Orlicz-Brunn-Minkowski inequalities. These inequalities are based on the Orlicz-addition of star bodies, and are thought to be the heart of the dual Orlicz-Brunn-Minkowski Theory. Finally, I will also mention the dual Orlicz-Minkowski problem, which is one of the hot topics in Brunn-Minkowski theory recently.

2010 MSC: Primary 52A20; Secondary 53A15.

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Abstracts in Low-dimensional Topology

Conjugacy stability of braids and spherical Artin-Tits groups

Cisneros de la Cruz, Bruno Aarón, Calvez Matthieu and Cumplido María

CONACyT – Instituto de Matemáticas de la UNAM, México
Email: bruno@im.unam.mx

Juan González-Meneses proved that geometric embeddings of braid groups do not merge conjugacy classes, i.e. parabolic subgroups of Artin braid groups are conjugacy stable. In this work we give a complete classification of conjugacy stable parabolic subgroups of Artin-Tits groups of spherical type. This answers a question posed by Ivan Marin.

2010 MSC: Primary 57M25; Secondary 20F36.

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A stably trivial link

Akio Kawauchi

Osaka City University, Japan
Email: kawauchi@sci.osaka-cu.ac.jp

It is shown that a handle-irreducible summand of every stably trivial surface-link is only a trivial 2-link. This means that a stably trivial surface-link is a trivial surface-link. By combining this result with an old result of F. Hosowaka and the speaker that every surface-knot with infinite cyclic fundamental group is a stably trivial surface-knot, it is concluded that every surface-knot with infinite cyclic fundamental group is a trivial (i.e., an unknotted) surface-knot.

2010 MSC: Primary 57Q45; Secondary 57N13.

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[3] A. Kawauchi, Ribbonness of a stable-ribbon surface-link, II. General case, preprint. http://www.sci.osaka-cu.ac.jp/ kawauchi/SRibbonSLinkII(GeneralCase).pdf

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Mapping classes are almost determined by their finite quotient actions

Yi Liu

Beijing International Center for Mathematical Research, China
Email: liuyi@bicmr.pku.edu.cn

For any closed surface, we say that two mapping classes are procongruently conjugate if they induce conjugate actions on the outer automorphism group of the profinite completion of the surface group. In this talk, I will sketch a proof of the following result: Every procongruent conjugacy class contains only finitely many conjugacy classes of mapping classes.

2010 MSC: Primary 57M50; Secondary 57M10, 30F40.

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[1] Yi Liu, Mapping classes are almost determined by their finite quotient actions, preprint, 42 pages, arXiv:1906.03602

On smooth fake projective planes

Jongil Park and Ju A Lee

Seoul National University, Republic of Korea
Email: jipark@snu.ac.kr

Since G. Prasad and S.-K.Yeung [2] proved that there exist only finitely many (in fact, exactly 100) fake projective planes in complex category, it has been an intriguing question how many smooth and symplectic fake projective planes exist in smooth and symplectic category respectively. A *smooth/symplectic fake projective plane* is a smooth/symplectic 4-manifold whose betti numbers are the same as those of $\mathbb{C}P^2$ but not homeomorphic to $\mathbb{C}P^2$.

In this talk I'd like to review known results on fake projective planes in complex category and I'll provide one way [1] how to construct an infinite family of smooth fake projective planes in smooth category. If a time is allowed, I'll also discuss the same problem in symplectic category.

2010 MSC: Primary 14J29; Secondary 57N13

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Existence of a transverse universal knot

Jesus Rodriguez-Viorato

Center for Research in Mathematics (CIMAT), Mexico
Email: jesusr@cimat.mx

We prove that there is a knot k transverse to ξ_{std}, the tight contact structure of S^3, such that every contact 3-manifold (M, ξ) can be obtained as a contact covering branched along k. By contact covering we mean a map $\varphi : M \to S^3$ branched along k such that ξ is contact isotopic to the lifting of ξ_{std} under φ.

2010 MSC: Primary: 53D10. Secondary: 53D15, 57R17.

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New quantum invariants of genus 2 handlebody-2-tangles

Sukuse Abe

Osaka City University, Japan
Email:k9250979@kadai.jp

We define a new quantum $U_q(\mathfrak{g})$ invariant of genus 2 handlebody-2-tangles, in the sense of Carmen Caprau [1], for any semi-simple lie algebra $\mathfrak{g}$. Every genus 2 handlebody-2-tangle is uniquely represented by a theta curve in $\mathbb{R}^2 \times [0, 1]$. The invariant is defined by using the associated link for the theta curve. As an application, we give a pair of distinct genus 2 handlebody-2-tangles whose exterior are homeomorphic to each other.

2010 MSC: Primary 57M27; Secondary 57M25.

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Dirichlet domains for one-cone torus bundles

Hirotaka Akiyoshi

Osaka City University, Japan
Email: akiyoshi@sci.osaka-cu.ac.jp

The combinatorial structures of Ford domains for once-punctured torus Kleinian groups are characterized by Jorgensen [1] (cf. [2]). A similar characterization is also obtained for a certain class of one-cone torus bundles in [3]. In this talk, we study Dirichlet domains for a certain class of one-cone torus bundles and see their combinatorial structures.

2010 MSC: Primary 57M50; Secondary 52A15.

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On Relation between quandle extension and group extension

Yongju Bae, Scott Carter and Byeorhi Kim

Kyungpook National University, Republic of Korea
Email: ybae@knu.ac.kr

Joyce and Matveev showed that a group G can be seen as a quandle $(G, \triangleleft_\zeta)$ by defining a quandle operation $\triangleleft_\varphi$ on G by using a group automorphism φ on G.

In this talk, we show a relationship between group extensions of a group G and quandle extensions of the quandle $(G, \triangleleft_\zeta)$. In fact, there exists a group homomorphism from $H^2_{\mathrm{gp}}(G; A)$ to $H^2_{\mathrm{q}}((G, \triangleleft_\zeta); A)$.

Also, we show a relationship between quandle extensions of a quandle Q and quandle extensions of the inner automorphism group $\mathrm{Inn}(Q)$ of Q. Indeed, there exists a group homomorphism from $H^2_{\mathrm{q}}(Q; A)$ to $H^2_{\mathrm{q}}((\mathrm{Inn}(Q), \triangleleft_\zeta); A)$.

2010 MSC: Primary 57M25; Secondary 57M27.

Quasipositive links and electromagnetism

Benjamin Bode

Osaka University, Japan
Email: b-bode@cr.math.sci.osaka-u.ac.jp

An electromagnetic field is a function $\mathbf{F}: \mathbb{R}^3 \to \mathbb{C}^3$. Such a field can contain knots and links in different ways, for example as flow lines of the electric field $\mathbf{E} = \mathrm{Re}(\mathbf{F})$, as flow lines of the magnetic field $\mathbf{B} = \mathrm{Im}(\mathbf{F})$ [3], or as the null lines $\mathbf{F}^{-1}(0,0,0)$ [2]. We are particularly interested in links that are stable for all time. That is, as the field $\mathbf{F}$ evolves according to Maxwell's equations, it contains a given link for all time. In this talk, I am going to explain how the construction of such electromagnetic fields can be related to holomorphic polynomials and thus to quasipositive links, which are given by the transverse intersection of a complex plane curve with a 3-sphere. I will illustrate how every link in S^3 can be constructed as a sublink of a quasipositive link and eventually as a subset of a stable null link of an electromagnetic field [1].

2010 MSC: Primary 57M25; Secondary 78A25.

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Skein modules: The braid approach

Ioannis Diamantis

China Agricultural University, China
Email: ioannis.diamantis@hotmail.com

In this talk we will develop an algebraic approach in the computation of *skein modules* of 3-manifolds, which are quotients of free modules over ambient isotopy classes of knots and links in the 3-manifolds by properly chosen skein relations. Our primary motivation is the computation of skein modules via a homogeneous way.

We start by providing algebraic mixed braid classification of links in any closed, connected and oriented 3-manifold M obtained by rational surgery along a framed link in S^3. We do this by representing M by a closed framed braid in S^3 and links in M by closed mixed braids in S^3 and we formulate the algebraic braid equivalence in terms of the mixed braid groups $B_{m,n}$, using cabling and the parting and combing techniques for mixed braids. A new isotopy move, called *band move* on the level of mixed links and *braid band move (bbm)* on the level of mixed braids, is also introduced and is the analogue of the second Kirby move. Our results set a homogeneous algebraic ground for studying links in 3-manifolds and in families of 3-manifolds using computational tools and our setting is appropriate for constructing Jones-type invariants for links in families of 3-manifolds via quotient algebras of the mixed braid groups $B_{m,n}$, as well as for studying skein modules of 3-manifolds, since they provide a controlled algebraic framework and much of the diagrammatic complexity is absorbed into the proofs.

Towards that end, we will present recent results on the HOMFLYPT skein module of the solid torus ST and the lens spaces $L(p,q)$, using braids and the generalized Iwahori-Hecke algebra of type B, and finally, we will also present recent results on the Kauffman bracket skein module of ST and $L(p,q)$ via the Temperley-Lieb algebra of type B.

2010 MSC: Primary 57M27; Secondary 57M25.

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The knots $k(\ell, m, n, p)$ and $(1,1)$-knots

Mario Eudave-Muñoz and José Frías

UNAM, Mexico
Email: mario@matem.unam.mx

We consider the family of knots $k(\ell, m, n, p)$ which have the property of being the only hyperbolic knots that have a half-integral surgery, that is, a Dehn surgery producing a manifold containing an incompressible torus [1], [2]. ℓ, m, n, p are integral parameters with some restrictions and always $n = 0$ or $p = 0$. It is well know that these knots have tunnel number one. We consider the question of whether these knots are $(1,1)$-knots. Remember that a knot is a $(1,1)$-knot if it has a 1-bridge presentation with respect to a standard torus.

First, we show that all the knots $k(\ell, m, 0, p)$ are $(1,1)$-knots. This is done by looking directly into the original tangle description of these knots. By means of computer calculations and using Yokota's inequality [3] about evaluations of the Jones polynomial at roots of unity, we show that infinitely many of the $k(\ell, m, n, 0)$ knots are not $(1,1)$-knots. We conjecture that all the knots $k(\ell, m, n, 0)$, $n \neq 0$, are not $(1,1)$-knots.

2010 MSC: Primary 57M25; Secondary 57M27.

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The Neuwirth conjecture and $(1,1)$-knots

José Frías

UNAM, Mexico
Email: frias@matem.unam.mx

Given a non-trivial knot $K \subset \mathbb{S}^3$, the Neuwirth conjecture states that there exists a closed surface F containing K as a non-separating curve such that every compressing disk for F intersects K in at least two points (see [3] for an extended exposition on the Neuwirth conjecture). We propose a new parametrization for $(1,1)$-knots different from the classical ones ([1]). We use this parametrization to construct explicit Neuwirth surfaces for most of the $(1,1)$-knots extending the constructions appearing in [2].

2010 MSC: Primary 57M25; Secondary 57N10.

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Twisted Alexander polynomial and matrix-weighted zeta function

Hiroshi Goda

Tokyo University of Agriculture and Technology, Japan
Email: goda@cc.tuat.ac.jp

The twisted Alexander polynomial is an invariant of the pair of a knot and its group representation. In this talk, we introduce a digraph obtained from an oriented knot diagram, which is used to study the twisted Alexander polynomial of knots. Then, we show that the inverse of the twisted Alexander polynomial of a knot may be regarded as the matrix-weighted zeta function which is a generalization of an Ihara–Selberg zeta function of a directed weighted graph [2].

2010 MSC: Primary 57M27; Secondary 05C50.

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New deformations on spherical curves and Ostlund Conjecture

Megumi Hashizume and Noboru Ito

Nara University of Education and OCAMI, Japan
Email: hashizume.megumi.y9@nara-edu.ac.jp

A spherical curve is the image of a generic immersion of a circle into a 2-sphere, and every spherical curve is transformed into the simple closed curve by a finite sequence of deformations, each of which is either one of type RI, type RII, or type RIII. In 2001([2]), Oestlund conjectured that deformations type RI and RIII are sufficient to describe a homotopy from any generic immersion from a circle into a plane to the standard embedding of the circle. In order to describe the equivalence class including the spherical curve with no double points, we introduce new certain deformations of type α and type β. We show that any two spherical curves are equivalent under deformations of type RI and type RIII up to ambient isotopy if and only if two RI-minimal spherical curves obtained from them are transformed each other by a finite sequence of deformations, each of which is either one of type RIII, type α, or type β. Here, any spherical curve is transformed into the minimal one that has no monogon, called the RI-minimal spherical curve, by successively applying deformations of type RI, each of which resolves a single double point [1].

This is a joint work with Noboru Ito (The University of Tokyo).

2010 MSC: Primary 57R42; Secondary 05C12, 57M99.

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Generalized quandle cocycle invariants and shadow quandle cocycle invariants

Atsushi Ishii and Kanako Oshiro

University of Tsukuba, Japan
Email: aishii@math.tsukuba.ac.jp

In [1], Carter et al. introduced several invariants by using generalized quandle 2-cocycles, which include an invariant obtained by summing up Boltzmann weights such as a quandle cocycle invariant. We call it a generalized quandle cocycle invariant. In this talk, we show that generalized quandle cocycle invariants are shadow quandle cocycle invariants.

2010 MSC: Primary 57M27; Secondary 57M25.

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On crosscap numbers of knots

Noboru Ito and Yusuke Takimura

The University of Tokyo, Japan
Email: Email:noboru@ms.u-tokyo.ac.jp

In this talk, we obtain the crosscap number of any alternating knots, and upper bounds of the crosscap number of knots (alternating or non-alternating). Our approach is closely related to band surgery. These estimates give two-sided volume bounds of alternating knots and an upper bound of hyperbolic volume of knots. This study improves the efficiency to choose a spanning surface given by a Kauffman state of an n-crossing projection from 2n possible state on Adams-Kindred minimal genus algorithm of spanning orientable or non-orientable surfaces of knots. We also con

rm that our equality gives the crosscap numbers of all the alternating knots with up to eleven crossings that include previously unknown 193 values. Information: After we submitted our manuscripts [1] (on 2018/10/12), [2] (on 2019/03/08), and [3] (on 2019/04/04), T. Kindred asked us (on 2019/05/17) to mention his working on crosscap number independently before his manuscript was submitted to arXiv [4].

2010 MSC: Primary 57M25, 57M27 ; Secondary 51M09

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A multivariable polynomial invariant of virtual links and cut systems

Naoko Kamada

Nagoya City Univeristy, Japan
Email: kamada@nsc.nagoya-cu.ac.jp

A cut system is a set of points (cut points) on semi-arcs of a virtual link diagram defined by H. Dye [1]. In [3], a construction of a normal virtual link from a virtual link is introduced by using cut points and some applications of the construction are given. An oriented cut system is an extension of cut system defined in [4]. A construction of a mod m almost classical virtual link from a virtual link is also introduced in [4] by using an oriented cut system. Here we introduce a polynomial invariant of a virtual link diagram using an oriented cut system. It turns out that the polynomial coincides with the multivariable polynomial invariant defined by L. Kauffman, H. Dye [2] and Y. Miyazawa [5]. We also discuss some properties of this invariant from a view point of our definition.

2010 MSC: Primary 57M25.

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Tensor products of quandles and classification of 1-handles attaching to surface-links

Seiichi Kamada

Osaka Univeristy, Japan
Email: kamada@math.sci.osaka-u.ac.jp

It is proved in Hosokawa and Kawauchi [2] that equivalence classes of 1-handles attached to oriented surface-knots or surface-links are completely determined from the homotopy classes of their cores. J. Boyle [1] classified 1-handles attached to oriented surface-knots by using the double cosets of the knot groups, and in Kamada [2] it is generalized to possibly non-orientable surface-knots. Here, we introduce the notion of a tensor product of quandles, and using it, we classify 1-handles attached to (oriented/non-orientable) surface-links.

2010 MSC: Primary 57Q45.

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Classification of ribbon 2-knots of 1-fusion with up to six crossings

Taizo Kanenobu

Osaka City University, Japan
Email: kanenobu@sci.osaka-cu.ac.jp

We consider the classification of ribbon 2-knots of 1-fusion with up to six crossings. The knot groups of such ribbon 2-knots have been classified by Takahashi [1]. In this talk we classify up to ambient isotopy. We use the Alexander polynomials, representations of the knot group to SL(2, C), and associated twisted Alexander polynomials.

2010 MSC: Primary 57Q45; Secondary 57M05.

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Calculating the Arf invariant of a proper link from a bicolored diagram

Kengo Kawamura

Osaka City University, Japan
Email: kengok@sci.osaka-cu.ac.jp

An oriented link L is said to be proper if the linking number $\mathrm{lk}(K, L \setminus K)$ is even for each component K in L. The Arf invariant $\mathrm{Arf}(L) \in \{0, 1\}$ of a link L is defined only when L is a proper link. It is known that there are several ways to calculate $\mathrm{Arf}(L)$, e.g. using Seifert forms, the polynomial invariants, local moves, and 4-dimensional techniques. In this talk, we introduce a certain way to calculate $\mathrm{Arf}(L)$.

2010 MSC: 57M25; 57M27.

Crossing number of an alternating 3-braid knot and canonical genus of its Whitehead double

Sang Youl Lee and Hee Jeong Jang

Pusan National University, Republic of Korea
Email: sangyoul@pusan.ac.kr

A conjecture proposed by J. Tripp (modified by T. Nakamura) states that the crossing number of any alternating knot coincides with the canonical genus of its Whitehead double. In the meantime, it has been proved that this conjecture is true for a large class of alternating knots including $(2, m)$ torus knots, 2-bridge knots, algebraic alternating knots and alternating pretzel knots. In this talk, we show that the conjecture is also true for a large class of alternating knots including all prime alternating 3-braid knots but not true for any alternating 3-braid knot which is the connected sum of two torus knots of type $(2, m)$ and $(2, n)$ with odd integers $m, n \geq 3$.

2010 MSC: Primary 57M25; Secondary 57M27.

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On 3-submanifolds of S^3 which admit complete spanning curve systems

Fengling Li

Dalian University of Technology, China
Email: fenglingli@dlut.edu.cn

Let M be a compact connected 3-submanifold of the 3-sphere S^3 with one boundary component F such that there exists a collection of n pairwise disjoint connected orientable surfaces $\mathcal{S} = \{S_1, \cdots, S_n\}$ properly embedded in M, $\partial\mathcal{S} = \{\partial S_1, \cdots, \partial S_n\}$ is a complete curve system on F. We call $\mathcal{S}$ a complete surface system for M, and $\partial\mathcal{S}$ a complete spanning curve system for M. In the present talk, we show that any complete spanning curve system for M is equivalent to $\partial\mathcal{S}$. This is joint work with Fengchun Lei and Yan Zhao.

Torus handle additions on boundary component of genus three

Chan-Palomo Luis Celso

Universidad Autónoma de Yucatán, México
Email: chpalomo@correo.uady.mx

Let M be a compact connected orientable 3-manifold and F be a boundary component of M with genus three. Denote by $M[\alpha]$ the 3-manifold obtained from M by attaching a 2-handle to M along a separating slope α on F. Let $\Delta(\alpha, \beta)$ denote the minimal geometric intersection number among all the curves representing the slopes. In a previous work [1] we proved that if M is simple, $M[\alpha]$ and $M[\beta]$ are toroidal then $\Delta(\alpha, \beta) \leq 10$. In this talk I will prove $\Delta(\alpha, \beta) \leq 8$ in some cases, this is work in progress.

2010 MSC: Primary 57M15; Secondary 57M50.

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Cocycles of G-Alexander biquandles and G-Alexander multiple conjugation biquandles

Shosaku Matsuzaki, Atsushi Ishii, Masahide Iwakiri, Seiichi Kamada, Jieon Kim and Kanako Oshiro

Takushoku University, Japan
Email: smatsuza@ner.takushoku-u.ac.jp

Biquandles and multiple conjugation biquandles (MCB) are algebras which are related to links and handlebody-links in 3-space. In this talk, we introduce MCB colorings of handlebody-links, and discuss cocycles of a certain class of biquandles and multiple conjugation biquandles, which we call G-Alexander biquandles and G-Alexander multiple conjugation biquandles, with a relationship with group cocycles.

2010 MSC: Primary 57M27; Secondary 57M25.

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Extensions of multiple conjugation quandles

Tomo Murao

University of Tsukuba, Japan
Email: t-murao@math.tsukuba.ac.jp

A quandle is an algebra whose axioms are motivated from knot theory. A linear/affine extension of a quandle can be described by using some maps called an (augmented) Alexander pair [1]. We can construct many invariants for knots by using (augmented) Alexander pairs. In this talk, we show that a linear/affine extension of a multiple conjugation quandle can be described by using some maps called an (augmented) MCQ Alexander pair, where a multiple conjugation quandle is an algebra whose axioms are motivated from handlebody-knot theory. As with quandles, a linear/affine extension of a multiple conjugation quandle plays an important role to construct handlebody-knot invariants.

2010 MSC: Primary 57M27; Secondary 57M25, 57M15.

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Semi-locally Alexander polynomials of knots

Takefumi Nosaka

Tokyo Institute of Technology, Japan
Email: nosaka@math.titech.ac.jp

From a certain setting in terms of semi-local rings, I introduced a generalization of the Alexander polynomial of a knot. I gave a tool of computing our Alexander polynomials of some knots and show their non-triviality. In this talk, I explain the definition and the computation.

The maximum and minimum genus of a multibranched surface

Makoto Ozawa and Mario Eudave-Muñoz

Komazawa University, Japan
Email: w3c@komazawa-u.ac.jp

Let X be a regular multibranched surface, and $\mathcal{P}(X)$, $\mathcal{S}(X)$ be the set of permutation systems, the set of slope systems respectively. For given $P \in \mathcal{P}(X)$ and $S \in \mathcal{S}(X)$, the neighborhood $N(X; P, S)$ of X with respect to P and S, which is a compact orientable 3-manifold with non-empty boundary, is defined up to homeomorphism. Let $\mathcal{M}$ be the set of closed orientable 3-manifolds. Then there exists an element $M \in \mathcal{M}$ and an embedding $N(X; P, S) \hookrightarrow M$. The Heegaard genus of M is denoted by $g(M)$. In [1], we defined the *maximum* and *minimum* genus of X as

$$\max g(X) = \max_{P \in \mathcal{P}(X), S \in \mathcal{S}(X)} \min_{M \in \mathcal{M}} \{g(M) \mid N(X; P, S) \hookrightarrow M\}$$

$$\min g(X) = \min_{P \in \mathcal{P}(X), S \in \mathcal{S}(X)} \min_{M \in \mathcal{M}} \{g(M) \mid N(X; P, S) \hookrightarrow M\}$$

In this talk, we give a lower bound for the maximum and minimum genus of a multibranched surface by the first Betti number and the minimum and maximum genus of the boundary of the neighborhood of it respectively. As its application, we show that the maximum and minimum genus of $G \times S^1$ is equal to twice of the maximum and minimum genus of G for a graph G respectively.

2010 MSC: Primary 57M25; Secondary 57M27.

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3-manifolds admitting locally large Heegaard splittings

Ruifeng Qiu

East China Normal University, China
Email: rfqiu@math.ecnu.edu.cn

In this talk, I will introduce the geometric and topological properties of 3-manifolds admitting locally large Heegaard splittings.

2010 MSC: Primary 57N10; Secondary 57M25.

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Exchange moves and non-conjugate braid representatives of links

Alexander Stoimenow and Reiko Shinjo

Gwangju Institute of Science and Technology, Republic of Korea
Email: stoimeno@stoimenov.net

The braid groups B_n were introduced in the 1930s in the work of Artin. An element in the braid group B_n is called an n-braid. Alexander related braids to knots and links in 3-dimensional space, by means of a closure operation. In that realm, it became important to understand the braid representatives of a given link. Markov's theorem relates these representatives by two moves, conjugacy in the braid group, and (de)stabilization, which passes between braid groups. Markov's moves and braid group algebra have become fundamental in Jones' pioneering work and its later continuation towards quantum invariants. Conjugacy is, starting with Garside's, and later many others' work, now relatively well group-theoretically understood. In contrast, the effect of (de)stabilization on conjugacy classes of braid representatives of a given link is in general difficult to control. Only in very special situations can these conjugacy classes be well described.

We are concerned with the question when infinitely many conjugacy classes of n-braid representatives of a given link occur. Birman and Menasco introduced a move called exchange move, and proved that it necessarily underlies the switch between many conjugacy classes of n-braid representatives of a given link. We will discuss several results when the exchange move is also sufficient for generating infinitely many such classes.

2010 MSC: 57M25 (primary), 20F36, 57M27 (secondary).

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Vassiliev knot invariants derived from cable Γ-polynomials

Hideo Takioka

Kyoto University, Japan
Email: takioka@math.kyoto-u.ac.jp

For coprime integers $p(>0)$ and q, the (p,q)-cable Γ-polynomial of a knot K is the Γ-polynomial of the (p,q)-cable knot of K, where the Γ-polynomial is the common zeroth coefficient polynomial of the HOMFLYPT and Kauffman polynomials. I will talk about Vassiliev knot invariants derived from cable Γ-polynomials.

2010 MSC: Primary 57M25; Secondary 57M27.

Generalized torsion and Dehn filling

Masakazu Teragaito and Tetsuya Ito, Kimihiko Motegi

Hiroshima University, Japan
Email: teragai@hiroshima-u.ac.jp

Most 3-manifold groups are torsion-free. However, if a 3-manifold group is not bi-orderable, then we conjecture that the group contains a generalized torsion. This conjecture has been verified for the fundamental groups of Seifert fibered manifolds, Solvable manifolds, cyclic branched covers of the figure-eight knots, and so on. In this talk, we examine the fundamental groups of the resulting manifolds obtained by Dehn surgery on knots.

2010 MSC: Primary 57M05; Secondary 57M07,57M25,06F15, 20F60.

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Dehn surgeries along twist knots and non trivial L-functions

Jun Ueki and Ryoto Tange

Tokyo Denki University, Japan
Email: uekijun46@gmail.com

Let F be a field with $\mathrm{char} F \neq 2$. We introduce an infinite family of representations $\rho : \pi_{J(2,2n)} \to \mathrm{SL}_2(F)$ of twist knot groups with trivial Reidemeister torsions, and prove that these ρ factor through $(3,1)$-Dehn surgeries. These ρ may admit non-trivial L-functions of the universal deformations over a CDVR O with the residue field F.

The main tools are the 2nd type Chebyshev polynomials $\varsigma_n(z)$ defined by $\varsigma_n(2\cos\theta) = \dfrac{\sin n\theta}{\sin\theta}$ and *their variants*. This work is based on [4, 3] by Nagasato and Tran, and is in the scope of our previous research [2] and [1].

2010 MSC: Primary 57M27; Secondary 57M25.

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Hurwitz action on tuples of permutations

Yoshiro Yaguchi

National Institute of Technology, Gunma College, Japan
Email: yaguchi-y@nat.gunma-ct.ac.jp

Hurwitz equivalence is an equivalent relationship in a direct product of a group (or a quandle), which is defined by using an natural action by a braid group on the direct product. In 1891, Hurwitz gave a system of representatives of Hurwitz equivalence on tuples of transpositions of a symmetric group. In this talk, we will give a system of representatives of Hurwitz equivalence on tuples of permutations of the symmetric group of degree 3. As its applications, we will also give a "standard form" of non-simple surface braids which are associated with non-simple branched covering of degree 3.

2010 MSC: Primary 20F36; Secondary 20F34.

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Studies of distance one surgeries on lens space $L(p, 1)$ and band surgeries on torus knot $T(2, p)$

Jingling Yang and Zhongtao Wu

The Chinese University of Hong Kong, China
Email: jlyang@math.cuhk.edu.hk

It has been well known that any closed, orientable 3-manifold can be obtained by Dehn surgery on a link in S^3. One of the most prominent problems in 3-manifold topology is to list all the possible lens spaces that can be obtained by a Dehn surgery along a knot in S^3, which has been solved by Greene. A natural generalization of this problem is to list all the possible lens spaces that can be obtained by a Dehn surgery from other lens spaces. Besides, considering surgeries between lens spaces is also motivated from DNA topology. In this talk, we will discuss distance one surgeries between lens spaces $L(p, 1)$ with $p \geq 5$ prime and lens spaces $L(n, 1)$ for $n \in \mathbb{Z}$, correspondingly band surgeries from $T(2, p)$ to $T(2, n)$, by using Heegaard Floer d-invariant. This is a joint work with Zhongtao Wu.

2010 MSC: 57M25.

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Counting conjugacy classes of groups with contracting elements

Ilya Gekhtman and Wenyuan Yang

Peking University, China
Email: wyang@math.pku.edu.cn

In this talk, we shall derive an asymptotic formula for the number of conjugacy classes of elements for a class of statistically convex-cocompact actions with contracting elements. Denote by $\mathcal{C}(n)$ (resp. $\mathcal{C}'(n)$) the set of (resp. primitive) conjugacy classes of pointed length at most n. The main result is an asymptotic formula as follows:

$$\sharp\mathcal{C}(n) \asymp \sharp\mathcal{C}'(n) \asymp \frac{\exp(\omega(G))}{n}.$$

As a consequence of the formulae, the conjugacy growth series is transcendental for all non-elementary relatively hyperbolic groups, graphical small cancellation groups with finite components. This is a joint work with Ilya Gekhtman (U. Toronto).

2010 MSC: Primary 20F65, 20F67.

Rigidity of matrix group actions on $\mathrm{CAT}(0)$ spaces with possible parabolic isometries

Shengkui Ye

Xi'an Jiaotong-Liverpool University, China
Email: Shengkui.Ye@xjtlu.edu.cn

It is well-known that $\mathrm{SL}_n(\mathbf{Q}_p)$ acts without fixed points on an $(n-1)$-dimensional $\mathrm{CAT}(0)$ space (the affine building). We prove that $n-1$ is the smallest dimension of $\mathrm{CAT}(0)$ spaces on which matrix groups act without fixed points. Explicitly, let R be an associative ring with identity and $E'_n(R)$ the extended elementary subgroup. Any isometric action of $E'_n(R)$ on a complete $\mathrm{CAT}(0)$ space X^d of dimension $d < n-1$ has a fixed point. Similar results are discussed for automorphism groups of free groups.

On the Cheeger-Chern-Simons invariant of a 3-manifold

Seokbeom Yoon and Jinsung Park

Korean Institute for Advanced Study, Republic of Korea
Email: sbyoon15@kias.re.kr

The Cheeger-Chern-Simons invariant, also known as the complex volume, is one of the well-studied invariants in hyperbolic geometry. Motivate by the work of Marche ([1]), I would like to extend it to an $\mathrm{SL}(n,\mathbb{C})$-representation of a 3-manifold possibly with tori boundary. Also, extending the work of Garoufalidis-Thurston-Zickert ([2]), I would like to introduce a notion of deformed Ptolemy coordinates and give a simplicial way to compute the Cheeger-Chern-Simons invariant.

2010 MSC: Primary 57M27; Secondary 57M25.

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Comparison of minimal equal lengths in a once punctured torus running over its relative Teichmuller space

Ying Zhang and Da Lei

Soochow University, China
Email: yzhang@suda.edu.cn

We consider the problem of minimizing the equal length of a pair of simple closed geodesics of given topological type in a once punctured hyperbolic torus with fixed geometric boundary data as the torus runs over its relative Teichmuller space. For certain specific pairs of curves with symmetry, we are able to determine the minimizing torus and hence the minimal length. It is natural to compare the minimal lengths for inequivalent pair of the same intersection number. As computer experiments show, there is a conjecture that the specific pair of slopes $(1/0, 1/n)$ has its minimal length smaller than any other pair of slopes $(1/0, m/n)$, regardless of the geometric boundary data. In the asymptotic case where the geometric boundary of the torus is a conic point and the cone angle is approaching 2π, we obtain a result which is stronger than the conjecture just mentioned.

2010 MSC: Primary 57M50; Secondary 30F45.

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Mapping class group of a Heegaard splitting

Yanqing Zou and Ruifeng Qiu

Dalian Minzu University, China
Email: yanqing@dlnu.edu.cn

It is known that every orientable compact 3-manifold admits a Heegaard splitting. So we can study 3-manifolds by Heegaard splittings.

We will introduce a locally large distance at least 2 Heegaard splitting and prove that its mapping class group is finite. Moreover, there is a nonhyperbolic 3-manifold so that its mapping class group is finite.

2010 MSC: Primary 57M27; Secondary 57M50.

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Completeness and injectivity

Soichiro Fujii

Kyoto University, Japan
Email: s.fujii.math@gmail.com

In 1967, Banaschewski and Bruns [2] showed that a poset is a complete lattice iff it is injective, i.e., any monotone map into it can be extended along an embedding. In 1956, Aronszajn and Panitchpakdi [1] showed that a metric space is hyperconvex iff it is injective, and in 2012, Kemajou, Künzi and Otafudu [4] proved its variant for quasi-metric spaces. In this talk, I will present a theorem that unifies the theorems of Banaschewski and Bruns, and of Kemajou, Künzi and Otafudu: I will show that for any quantale $\mathcal{Q}$, a $\mathcal{Q}$-category is skeletal and complete iff it is injective with respect to fully faithful $\mathcal{Q}$-functors. The key construction is that of Isbell completion of $\mathcal{Q}$-categories, which generalises both MacNeille completion of posets and directed tight spans of quasi-metric spaces. This talk is based on [3, Sections 6 and 7].

2010 MSC: Primary 18D20; Secondary 06F07, 54E35.

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Quantale-valued topological spaces via closure and convergence

Hongliang Lai and Walter Tholen

Sichuan University, China
Email:hllai@scu.edu.cn

For a quantale V we introduce V-valued topological spaces via V-valued point-set-distance functions. When V is completely distributive, we characterize them in terms of both, so-called closure towers and ultrafilter convergence relations. When V is the two-element chain 2, the extended real half-line $[0, \infty]$, or the quantale $\mathbf{\Delta}$ of distance distribution functions, the

general setting produces known and new results on topological spaces, approach spaces, and the probabilistic approach spaces, as well as on their functorial interactions with each other.

2010 MSC: 54A20; 54B30

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SI-convergence in T_0 spaces

Jing Lu and Bin Zhao

Xi'an Jiaotong University, China
Email: lujing0926@126.com

In an invited talk at the Sixth International Symposium on Domain Theory, Lawson emphasized the need to develop the core of domain theory directly in T_0 spaces instead of posets. Towards this new direction, recently, motivated by the definition of the Scott topology on posets, Zhao and Ho (see [1]) introduced a method of deriving a new topology τ_{SI} from a give one τ, in a similar way as one derives the Scott topology on a poset from the Alexandroff topology on the poset. They called this topology the irreducibly-derived topology (or simply, SI-topology). Their working principle uses irreducible sets as the topological counterparts of directed sets. On the basis of this principle, Zhao, Lu and Wang (see [2]) introduced irreducible convergence in T_0 spaces, which can be seen as topological counterpart of lim-inf-convergence in posets, and then they gave a sufficient and necessary condition for irreducible convergence to be topological in T_0 spaces. It is proved that the irreducible topology is finer than the SI-topology. In order to know which convergence structure induces the SI-topology of a given T_0 space, Andradi, Shen, Ho and Zhao (see [3]) introduced the SI-convergence in T_0 spaces and proved that the topology raising from SI-convergence is just the SI-topology. Naturally, they posed a question as follows:

Problem: whether can find a complete characterization of T_0 spaces for the SI-convergence being topological?

In this talk, we introduce the notion of I^*-continuity, and prove that the SI-convergence in a T_0 space is topological if and only if the T_0 space is an I^*-continuous space. Thus we give a complete characterization of T_0 spaces for the SI-convergence being topological.

2010 MSC: Primary 54D10; Secondary 06B30; 06B35.

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Core-compactness of Smyth powerspaces

Zhenchao Lyu and Xiaodong Jia

Sichuan University, China
Email: zhenchaolyu@gmail.com

Given a topological space X, the *Smyth powerspace* $Q(X)$ is the set of compact saturated subsets of X with the *upper Vietoris topology*. In domain theory, the Smyth powerspace coincides with the Smyth powerdomain for continuous domains with the Scott topology, where the latter construction is used in modelling non-deterministic computation, see for example [10], [4]. We prove that the Smyth powerspace $Q(X)$ of a topological space X is core-compact if and only if X is locally compact. As a straightforward consequence we obtain that the Smyth powerspace construction does not preserve core-compactness generally. Consonance is another interesting topological property. A core-compact topological space is locally compact iff it is consonant. We prove that the Smyth powerspace $Q(X)$ of a topological space X is consonant implies X is consonant. And the converse is also true under some assumptions on X.

2010 MSC: 54B20, 06B35; 06F30.

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Concept lattices via quantaloid-enriched categories

Lili Shen

Sichuan University, China
Email: shenlili@scu.edu.cn

The theory of quantaloid-enriched categories has deeply impacted the study of formal concept analysis (FCA) and rough set theory (RST) by providing them with a general categorical framework. In this talk we give a brief introduction to Isbell adjunctions and Kan adjunctions between quantaloid-enriched categories, whose fixed points constitute the categorical version of concept lattices in FCA and RST. Based on a series of joint works with Javier Gutiérrez García, Hongliang Lai, Yuanye Tao, Walter Tholen and Dexue Zhang.

2010 MSC: Primary 18A40; Secondary 18D20.

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Some recent advances in non-Hausdorff topology

Xiaoquan Xu

Minnan Normal University, China
Email: xiqxu2002@163.com

In this talk, we will briefly review the main advances in non-Hausdorff topology, with emphasis on the recent progress in the theory of well-filtered T_0 spaces and sober spaces, and list a number of remaining problems.

Lower topology poset models of T_1 spaces

Shen Chong and Xi Xiaoyong Zhao Dongsheng

Beijing Institute of Technology, China
Email: shenchong0520@163.com

The lower topology $\omega(P)$ on a poset P is the topology generated by the sets of the form $P\setminus\uparrow x$, $x \in P$. A lower topology poset model (a poset LT-model for short) of a topological space X is a poset P such that the subspace $\mathrm{Max}(P)$ of the lower topological space $\Omega P = (P, \omega(P))$ is homeomorphic to X. The main results of this paper are as follows:

(1) Every T_1 space has a bounded complete algebraic poset LT-model.

(2) Every T_1 space has an algebraic dcpo LT-model.

(3) A T_1 space is compact iff it has a bounded complete (algebraic) dcpo LT-model.

(4) A T_1 space is second countable iff it has an ω-algebraic poset.

2010 MSC: .Primary 06B30; Secondary 54A10.

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Digital topological rough set structures and their applications

Sang-Eon Han

Chonbuk National University, Republic of Korea
Email:sehan@jbnu.ac.kr

The present talk introduces locally finite covering rough set theory and its applications [1,2,3]. Given a locally finite covering approximation (LFC-, for brevity) space $(U, \mathbf{C})$, for a subset X of the universe U, two kinds of neighborhood systems derived from the covering $\mathbf{C}$ are considered [1,2], where all sets U, $\mathbf{C}$ and $X(\subseteq U)$ need not be finite. Next, we consider two topologies on U, Alexandroff and quasi-discrete topologies generated by these neighborhood systems as bases, respectively.

Using these notions and systems, we study four types of rough set structures associated with ome digital topological structures. Further, we investigate various properties of these operators which can facilitate the study of applied science including digital geometry, information geometry, computer vision, pattern recognition, image processing, and so on. In order to accomplish this work, we use some tools such as granulations [7], neighborhood systems [5], typical Pawlak's tools[6], related topologies [4,7,8] rough set structures associated with some digital topologies.

2010 MSC: Primary 03E47; Secondary 03E75, 54A05, 54A10, 68T30, 97E60

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Investigation of 'flash crash" via topological data analysis (TDA)

Wonse Kim, Younng-Jin Kim, Woong Kook and Gihyun Lee

Seoul National University, Republic of Korea
Email:aquinasws@snu.ac.kr

There is by now a quite extensive literature on applications of TDA. But there are only a few literatures on applications of TDA to financial data, including the recent result of Gidea-Katz (2018). Interestingly, Gidea-Katz (2018) showed that a stock market crash can be foreseen via TDA by utilizing daily US stock market indices data (e.g., S&P 500, DJIA, NASDAQ, and Russell 2000). However, the Flash Crash on 6 May 2010 showed that the market can be substantially destabilized in as little as about 30 min. Since the Flash Crash, analyses of market crash of the intraday-horizon has also become important parts of the study of market crash. In this talk, I will demonstrate that the TDA methodology based on Gidea-Katz (2018) can be used in forecasting short term market crash such as Flash Crash.

2010 MSC: Primary 91B24; Secondary 55N35.

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On some categories of metric-type spaces based on extended t-conorms

Alexander Šostak and Tarkan Öner

University of Latvia, Latvia
Email: sostaks@lumii.lv

An important class of spaces was introduced by I.A. Bakhtin [1] and independently rediscovered by S. Czerwik [2], [3] under the name "b-metric spaces". b-metric spaces generalize "classic" metric spaces by replacing in the definition of a metric the triangularity axiom $d(x,z) \leq d(x,y) + d(y,z)$ for all $x,y,z \in X$ with a more general axiom $d(x,z) \leq k \cdot (d(x,y) + d(y,z))$ for all $x,y,z \in X$ where $k \geq 1$ is a fixed constant. The class of b-metric spaces includes such interesting and important for applications cases as $d : \mathbb{R} \times \mathbb{R} \to \mathbb{R}^+$ ($\mathbb{R}^+ = [0,\infty)$) defined by $d(x,y) = |x-y|^2$ or $d : C[a,b] \times C[a,b] \to \mathbb{R}^+$ defined by $d(f,g) = \int_a^b (f(x) - g(x))^2 \mathrm{d}x$.

Recently there were published several papers where the induced topology of b-metric spaces was applied. Unfortunately, in most papers the authors assumed that the open balls $B(a,r) = \{x \in X \mid d(a,x)\}$ are "really open", that is open in the induced topology. However, as it was noticed in [5], see also [6], [7] generally it is not true.

To "remedy" this "shortage" of b-metric spaces, Kirk and Shahzad [5] introduced the class of strong b-metric spaces or sb-metric for short. In the definition of an sb-metric space the third (triangular) axiom is given by the inequality $d(x,y) \leq d(x,z) + kd(z,y)$ for some fixed $k \geq 1$. Obviously the class of sb-metric spaces lies in between the class of metric and the class of b-metric spaces. As shown by Kirk and Shahzad [5], in sb-metric spaces open balls are "really" open, that is, open in the induced topology. Thanks to this fact, sb-metric spaces have many useful properties common with ordinary metric spaces. Being mainly interested in sb-metric spaces in this talk, we enlarge the framework of our research by replacing the addition "+" in the third axiom with an extended continuous t-conorm $\oplus$ (see e.g. [4]), satisfying certain conditions. In the result we come to categories of $\oplus$-**Metr**, $\oplus$-**b-Metr** and $\oplus$-**sb-Metr** spaces. We discuss some properties of these categories, in particular prove the existence of products and coproducts. Some examples of $\oplus$-sb-metric spaces, specifically of sb-metric spaces (the lack of which the authors of [5] complained) will be given.

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Fuzzy relational morphological spaces: basic properties and topological interpretation

A. Šostak and I. Uljane

University of Latvia, Latvia
Email: sostaks@latnet.lv

Mathematical morphology has its origins in geological problems centered in the processes of erosion and dilation. The founders of mathematical morphology are engineers G. Matheron [5] and J. Serra [6]. At present mathematical morphology can be characterized as a mathematical theory that focuses on studying transformations, especially non-linear, of geometrical objects. The central concepts of study in mathematical morphology are erosion and dilation. Mathematical morphology has applications in different areas of theoretical and applied science, in particular in pattern recognition, image processing, etc. Problems related to mathematical morphology were studied by many authors, see e.g. [1], [2], [3], [4], just to mention some of them. There are different approaches to the subject of mathematical morphology. We stick here on the so called L-fuzzy relational mathematical morphology, see, e.g. [4].

AL-spaces and algebraic L-domains

Longchun Wang, Xiangnan Zhou and Qingguo Li

Hunan University, China
Email: longchunw@163.com

Algebraic L-domains has nice mathematical properties, for example, they form one of maximal cartesian closed full subcategories of the category of algebraic domains with Scott-continuous functions. There are many approaches to characterizing algebraic L-domains, such as information systems, logic, formal topologies, etc.

We define a notion of the AL-space which is a non-T_0 topological space. Our goal here is to develop an objective characterization of algebraic L-domains via AL-spaces in purely topological terms. Main results are:

(1) A dcpo is an algebraic L-domain if and only if it is an AL-space when it equipped with the Scott topology.

(2) The collection of T-points of an AL-space ordered by set inclusion is an algebraic L-domain, and every algebraic L-domain can be obtained by this way up to isomorphism.

(3) The relationship of Lawson compact algebraic L-domains and a class of subspaces of AL-spaces is depicted.

2010 MSC: Primary 06B35; Secondary 54F65.

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The monad of saturated prefilters

Hongliang Lai, Dexue Zhang, Gao Zhang

Sichuan University, China
Email: dxzhang@scu.edu.cn

Let & be a continuous t-norm on the unit interval $[0, 1]$ such that the implication operator is continuous at each point off the diagonal of $[0, 1] \times [0, 1]$. Then it is proved that

- assigning each set X to the set of all saturated prefilters on X gives rise to a monad;
- assigning each set X to the set of all $\top$-filters on X gives rise to a monad.

These two monads are useful in the study of fuzzy ordered sets and fuzzy topological spaces.

2010 MSC: Primary 18C15; Secondary 18B30.

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Generalizations of Martin's Axiom, square and a forcing axiom failure

David Asperó

University of East Anglia, UK
Email: davidaspero@icloud.com

I will present consistent forcing axioms extending Martin'ss Axiom at λ for arbitrary λ, will prove that one of them implies $\square_{\omega_1}$, and will use this result to show that a natural extension of Martin's Axiom is inconsistent.

Some more applications of ω_1-strongly compact cardinals in General Topology

Joan Bagaria

ICREA and Universitat de Barcelona, Catalonia
Email: bagaria@ub.edu

An uncountable cardinal κ is called ω_1-*strongly compact* if every κ-complete filter on any set I can be extended to an ω_1-complete ultrafilter on I. ω_1-strongly compact cardinals are important for the study of compactness in General Topology, a prominent example being that the product of Lindelöf spaces is κ-Lindelöf if and only if κ is greater than or equal to the least ω_1-strongly compact cardinal [1, 2]. We shall present some results obtained in a joint work with S. Gomes da Silva [3], yielding more applications of ω_1-strongly compact cardinals in the context of either consistency or reflection results in General Topology, focusing on issues related to normality. In particular, we show that the existence of an ω_1-strongly compact cardinal provides a new upper bound for the consistency strength of the statement "All normal Moore spaces are metrizable". We also establish a compactness theorem for normality (i.e. reflection of non-normality) in the realm of first countable spaces, using the least ω_1-strongly compact cardinal.

2010 MSC: Primary 54D15; Secondary 03E55, 54A35.

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Some constructions involving inverse limits

Aleksander Blaszczyk

University of Silesia, Poland
Email:ablaszcz@math.us.ed.pl

Inverse limits appears in several branches of topology including theory of continua, general topology and set-theoretic topology. It also appears in the theory of Boolean algebras as chains of subalgebras. I will briefly survey the most influential topological constructions using inverse limits. Then I will present new constructions using inverse limits. In particular there will be shown that every compact space is an irreducible image of a compact F-space of the minimal weight. Concerning a problem of Arhangel'skii there will be proved that under some additional set-theoretical assumptions every compact first countable space is an image of a first countable zero-dimensional compact space.

2010 MSC: Primary 54B35; Secondary 54A35.

On equivalence relations generated by Cauchy sequences in countable metric spaces

Longyun Ding

Nankai University, China
Email: dinglongyun@gmail.com

Let $\mathbb{X}$ be the set of all metrics on ω, and let $\mathbb{X}_{\rm cpt}$ be the set of all metrics r on ω that the completion of (ω, r) is compact. We define Cauchy sequence equivalence relation $E_{\rm cs}$ on $\mathbb{X}$ as: $rE_{\rm cs}s$ iff the set of Cauchy sequences in (ω, r) is same as in (ω, s). We also denote $E_{\rm csc} = E_{\rm cs} \restriction \mathbb{X}_{\rm cpt}$.

We show that $E_{\rm cs}$ is a $\mathbf{\Pi}^1_1$-complete equivalence relation, while $E_{\rm csc}$ is a $\mathbf{\Pi}^0_3$ equivalence relation. We also show that $E_{\rm csc}$ is Borel bireducible to an orbit equivalence relation. Furthermore, we tried to find out the Borel reducibility between $E_{\rm csc}$ and some benchmark equivalence relations. For instance, we show that $=^+$ and $\mathbb{R}^\omega/c_0$ are Borel reducible to $E_{\rm csc}$, and E_1 is not.

Restrictions of $E_{\rm csc}$ on some special subsets of $\mathbb{X}_{\rm cpt}$ are also considered.

This is a joint work with Kai Gu.

Set-theoretic principles which imply that the continuum is fairly large

Sakaé Fuchino

Kobe University, Japan
Email: fuchino@diamond.kobe-u.ac.jp

The continuum can be fairly large: it can be a weakly Mahlo cardinal or can even carry an $< \aleph_1$-saturated normal ideal.

We consider some set-theoretic principles mainly in form of reflection principles which either imply or are compatible with the fairly large continuum. Part of the results presented here appear in the following joint works with André Ottenbereit Maschio Rodriques and Hiroshi Sakai.

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Extend version of the paper: http://fuchino.ddo.jp/papers/SDLS-II-x.pdf

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Preserving selection principle properties

Jialiang He, Boaz Tsaban and Shuguo Zhang

Sichuan Univeristy, China
Email: jialianghe@scu.edu.cn

Let (X, τ) be a topology space in ground model V. In $V^{\mathbb{P}}$(forcing extension by $\mathbb{P}$), we consider a new topology $\tau^{\mathbb{P}}$ on X:

$$\tau^{\mathbb{P}} = \{\bigcup S : S \subseteq \tau\}$$

Let Q is some topology property. We call Q *is is preserved by* $\mathbb{P}$ if $(X, \tau^{\mathbb{P}})$ still has Q for each (X, τ) has Q.

Selective topological covering properties form the kernel of selection principles. Such as, for families $\mathcal{A}$ and $\mathcal{B}$ of sets, let $\mathbf{S}_1(\mathcal{A}, \mathcal{B})$ be the statement: For each sequence of elements of the family $\mathcal{A}$, we can pick one element from each sequence member, and obtain an element of the family $\mathcal{B}$. When $\mathcal{A} = \mathcal{B} = \mathcal{O}$, the family of open covers of the ambient space, we obtain Rothberger's property.

M. Scheepers and F. Tall investigated the preservation results of selection principle in Cohen and Random forcing. Some question remains open, for example,

- Is non-Rothberger preserved by Random forcing?
- Does adding a single Cohen real convert each ground-model Lindelof of space to a Rothberger space?

We will answer these questions in this talk.

2010 MSC: Primary: 26A03, Secondary: 03E75, 03E17

Reflection principles down to the continuum and an application of mixed support iteration

André Ottenbreit Maschio Rodrigues

Kobe University, Japan
Email: ottenbreit@stu.kobe-u.ac.jp

This is a joint work with Sakaé Fuchino and Hiroshi Sakai.

Given some weak second order logic $\mathcal{L}$ and a regular cardinal κ, we define the Strong Downward Löwenheim-Skolem reflection on $\mathcal{L}$ down to $< \kappa$ (denoted by $\mathrm{SDLS}(\mathcal{L}, < \kappa)$) as the statement:

> For any structure $\mathfrak{A}$ of countable signature and cardinality $\geq \kappa$, there is a structure $\mathfrak{B}$ of cardinality $< \kappa$ such that $\mathfrak{B} \prec_{\mathcal{L}} \mathfrak{A}$.

In this talk, we discuss variations of this SDLS reflection principle related to stationary logics which shall be defined by introducing the second order quantifier $\mathrm{stat}\, X$, interpreted as "for stationary many X". We discuss the restrictions these principles impose on the size of the continuum.

Then, we use a mixed support iteration to construct, starting from a model with 2 supercompact cardinals, a model where the continuum is fairly large, all the cardinal invariants in Cichoń's diagram are equal to the continuum and a strong version of SLDS down to $\leq 2^{\aleph_0}$ holds simultaneously with a weaker version of SDLS down to $< 2^{\aleph_0}$.

Basis sizes of uncountable linear orders

Yinhe Peng

CAS, China
Email: pengyinhe@amss.ac.cn

Moore proved that the basis size of uncountable linear orders is consistent to be 5. It is well known that the basis can consistently have size 2^{ω_1}. We (jointly with Liuzhen) show that for any natural number $n > 5$, it is consistent to have a basis of size n. We also investigate the basis size for each type: Aronszajn type and real type.

Undefinability via topology

Clovis Hamel and Franklin D. Tall

University of Toronto, Canada
Email: f.tall@utoronto.ca

Definability is a subject of interest both in Set Theory and Model Theory. Definable objects are somehow simpler than undefinable ones. Of course "definable" itself needs to be defined, and that will depend on the language and the logic being used. A natural question popularized by W. T. Gowers is whether a notoriously pathological Banach space of Tsirelson is (explicitly) definable. This was recently solved negatively for first-order logic (and its continuous variant) by P. Casazza and J. Iovino. We realized that A. V. Arhangel'skiĭ's $\mathrm{C_p}$-theory (the study of the pointwise topology on spaces of continuous real-valued functions on a topological space) would enable us to greatly generalize the kinds of logics for which such undefinability results could be proved. In Moscow last year, we reported our preliminary results, which we have now further generalized. The key idea is to establish a connection between stability (a cornerstone of contemporary Model Theory which gives a bound on the number of types), definability, and double limit conditions, which, as in Analysis, assert that if certain double limits exist, the order in which we take them doesn't affect the result. This was known for first-order logic because of its compactness, but, using $\mathrm{C_p}$-theory, we prove that much weaker conditions suffice. In particular, we prove that Tsirelson's space is not even definable in the infinitary logic $\mathcal{L}_{\omega_1,\omega}$ (and in its continuous variant). Our topological methods are promising for extending other results in Model Theory beyond first-order logic.

Large cardinals as natural upper bounds on cardinal functions

Toshimichi Usuba

Waseda University, Japan
Email: usuba@waseda.jp

In this talk, we present some large cardinals, such as measurable cardinals and strongly compact cardinals, give natural upper bounds on cardinal functions of topological spaces. For instance, we show that the least ω_1-strongly compact cardinal is the precise upper bound on the Lindelöf degree of G_δ-topology of compact spaces.

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Some game-theoretic characterizations for rapid filter on ω

Zuoheng Li and Shuguo Zhang

Sichuan University, China
Email: zhangsg@scu.edu.cn

Let $\mathcal{F}$ is a filter on ω, $f \in \omega^\omega$, The game $G(\mathfrak{Fr}, [\omega]^{\leq f}, \mathcal{F})$ is played by two players ***ONE*** and ***TWO*** as follows: at stage $k < \omega$, ***ONE*** chooses $X_k \in \mathfrak{Fr}$ and ***TWO*** responds with

$s_k \in [X_k]^{\leq f(k)}$. At the end of the game, ***TWO*** wins the game if $\bigcup_{k\in\omega} s_k \in \mathcal{F}$. Otherwise, ***ONE*** wins.

$G(\mathfrak{Fr}, \omega, \mathcal{F})$ means that ***TWO*** responds with $n_k \in X_k$ and $G(\mathfrak{Fr}, [\omega]^{<\omega}, \mathcal{F})$ means that ***TWO*** responds with $s_k \in [X_k]^{<\omega}$.

Tomek Bartoszynski, Claude Laflamme and Marion Scheepers had obtained that $G(\mathfrak{Fr}, \omega, \mathcal{F})$ and $G(\mathfrak{Fr}, [\omega]^{<\omega}, \mathcal{F})$ are game-theoretic characterizations for Q-filters and meager filters respectively in [1, 2]. In this talk, we will show that there are some $f \in \omega^\omega$ such that $G(\mathfrak{Fr}, [\omega]^{\leq f}, \mathcal{F})$ are game-theoretic characterizations for rapid filters, and we will show other types of game-theoretic characterizations for rapid filters.

2010 MSC: Primary 03E05. Secondary 91A44.

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Absolutely strongly star-Hurewicz property with respect to an ideal

Sumit Singh, Brij K. Tyagi and Manoj Bhardwaj

University of Delhi, India
Email: sumiitkumaar405@gmail.com

A space X is said to be absolutely strongly star-$\mathcal{I}$-Hurewicz (ASS$\mathcal{I}$H) space if for each sequence $(\mathcal{U}_n : n \in \mathbb{N})$ of open covers of X and each dense subset Y of X, there is a sequence $(F_n : n \in \mathbb{N})$ of finite subsets of Y such that for each $x \in X$, $\{n \in \mathbb{N} : x \in St(F_n, \mathcal{U}_n)\} \in \mathcal{I}$, where $\mathcal{I}$ is the proper admissible ideal of $\mathbb{N}$. In this paper, we investigated the relationships among ASS$\mathcal{I}$H and related spaces and studied the topological properties of ASS$\mathcal{I}$H space. Our results improved some earlier results from [22, 23].

2010 MSC: Primary 54D20. Secondary 54B20.

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Several cardinal invariants associated to ideal γ-covers

Jaroslav Šupina

University of P.J. Safarik, Slovakia
Email: jaroslav.supina@upjs.sk

We consider γ-covers on a topological space with respect to ideals on natural numbers. Applying such covers in S_1 selection principles appearing in Scheepers diagram gives rise to possibly new principles. Our studies focus on their critical cardinalities (the minimal cardinality of a space not satisfying the principle).

We present combinatorial characterizations of studied critical cardinalities and we discuss their relation to standard invariants in Cichoń or van Douwen diagram.

Abstracts in Topology and Dynamical Systems

Ergodic theory of random anosov systems mixing on fibers

Zeng Lian

Sichuan University, China

Study on hyperbolic systems has a long history and quite a lot of remarkable results has been derived in field. In this talk, I will report our recent study on the dynamics of uniformly hyperbolic systems driven by an external force, of which we derived the existence of periodic structures and horseshoes. This is the joint work with Wen Huang and Kening Lu.

Random walks on groups acting on dendrites

Andrei Malyutin

St. Petersburg State University, Russia

In my talk, I will try to explain the concepts of random walks on groups and their Poisson-Furstenberg boundaries. Then we will discuss several results related to random walks on groups acting on trees, R-trees, dendrites and other tree-like spaces. In particular, I will present a theorem about convergent sequences of dendrite's points corresponding to paths of random walks and a theorem about mappings of Poisson-Furstenberg boundaries to the sets of dendrite's regular points and endpoints.

Some results about sensitivity on linear dynamical systems

Quanquan Yao and Peiyong Zhu

University of Electronic Science and Technology of China, China
Email: zpy6940@sina.com.cn

We study the sensitivity for two class of systems: non-autonomous discrete systems $(X, f_{1,\infty})$, where $f_{1,\infty} = (f_n)_{n=1}^{\infty}$ is a sequence of operators on a Frechet space X, and C_0-semigroups $(T_t)_{t\geq 0}$ defined on X. The equivalence between several forms of sensitivity are obtained for such systems.

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On coupling lemma and stochastic properties for one-dimensional expanding maps

Jianyu Chen

Soochow University, China

In this talk, we establish a coupling lemma for standard families in the setting of piecewise expanding interval maps with countably many branches. This method is particularly powerful for maps whose inverse Jacobian has low regularity and those who does not satisfy the big image property. The main ingredient of our proof is to estimate the average length of standard families under the operations of cutting, iterates and splitting in terms of the characteristic $\mathcal{Z}$ function.

We further conclude the existence of an absolutely continuous invariant measure, the exponential decay of correlations and the almost sure invariance principle (which is a functional version of the central limit theorem). The latter two stochastic properties hold for a large class of unbounded observables, due to our crucial assumption called Chernov's one-step expansion at q-scale.

This is a joint work with Hong-Kun Zhang and Yiwei Zhang.

Topological entropy and indecomposability of G-like continua

Hisao Kato

University of Tsukuba, Japan
Email: hkato@math.tsukuba.ac.jp

For a graph G we define a new notion of free tracing property by free G-chains on G-like continua and we show that a positive topological entropy homeomorphism f of a G-like continuum X admits a Cantor set Z in X and an indecomposable subcontinuum H of X satisfying the following conditions (1)-(4);
(1) Z has the free tracing property by free G-chains,
(2) H is the unique minimal subcontinuum of X containing Z and no two points of Z belong the same composant of H,
(3) any sequence $(z_1, z_2, ..., z_n)$ of points in Z is an IE-tuple of f, and
(4) f is Li-Yorke chaotic on Z.
Also we show that the property of free tracing property by free G-chains is a necessary and sufficient condition for the existence of indecomposable subcontinua in G-like continua.

2010 MSC: Primary 54H20; Secondary 54F15.

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Recent developments of chaos theory in topological dynamics

Jian Li

Shantou University, China
Email: lijian09@mail.ustc.edu.cn

In this talk, we will discuss some recent developments on chaos theory and will focus on the complexity of dynamical systems with positive topological entropy.

2010 MSC: Primary 37B05; Secondary 54H20, 37B40.

The singularities of special developable surfaces of frame curves in the 3-Euclidean space

Yanlin Li

Hangzhou Normal University, China

In this presentation, I will talk about the singularity properties of special developable surfaces of framed curves in Euclidean 3-space. Based on several work of S. Izumiya, G. Ishikawa and M. Takahashi, we first give the relationship between types of singularities of tangent developable surfaces of framed curves and frame curvature functions. That enables us to recognize the type of singularities of tangent developable surfaces of framed curves from the frame curvature functions of the frame curves. Secondly, combining some work of D. Mond and using the links of finitely determined ruled surfaces and their Gauss words, we give the classification and topological information of simple singularities of special developable surfaces. This is a joint work with Professor Zhigang Wang.

Margulis-Ruelle inequality for general manifolds

Gang Liao

Soochow University, China

We investigate the Margulis-Ruelle inequality for general Riemannian manifolds (possibly noncompact and with boundary) and show that this inequality always holds under integrable condition.

On topological properties of the Ellis semigroup

Y. Rodríguez-López, S. García-Ferreira, C. Uzcátegui

Universidade Federal de Minas Gerais, Brazil
Email: yackelin@mat.ufmg.br

A dynamical system (X, f) consists of a compact metric space X and a continuous function $f : X \to X$. A very important tool in the study of the topological behavior of a dynamical systems is its Ellis semigroup $E(X, f)$, defined as the pointwise closure of $\{f^n : n \in \mathbb{N}\}$ in the compact space X^X with composition of functions as its algebraic operation [1]. The Ellis semigroup is equipped with the topology inhered from the product space X^X. The continuity and discontinuity of the elements of $E(X, f) \setminus \{f^n : n \in \mathbb{N}\}$ were studied in the papers [2], [3], [5], [6] and [7]. Indeed, the authors have investigated the structure of the Ellis semigroup of a dynamical system and the topological properties of some of its elements, whose phase spaces are compact metrizable countable spaces (see [3], [4] and [5]). The main tool that have been used in all these investigations is the combinatorial properties of the ultrafilters on $\mathbb{N}$. Certainly, the Ellis semigroup can be described in terms of the notion of convergence with respect to an ultrafilter. Hence, our main purpose moves in two directions: The first one concerns the continuity and discontinuity of the elements of $E(X,f) \setminus \{f^n : n \in \mathbb{N}\}$ and the second one concerns about conditions for $E(X,f)$ to be homeomorphic to X. In this work we shall present recent results and examples in these directions.

2010 MSC: Primary 54H20, 54G20; Secondary 54D80.

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Minimal sets for amenable group actions on uniquely arcwise connected continua

Enhui Shi

Soochow University, China

An uniquely arcwise connected continuum is a continuum containing no simple closed curve. We show that every countable amenable group action on a uniquely arcwise connected continuum X must have a finite orbit; if X is further locally connected, then every minimal subset M of such actions is either finite or a Cantor set, and the restriction action to M is equicontinuous, which answered a question proposed by E. Glasner and M. Megrelishvili. This is a joint work with Professor Xiangdong Ye.

Densely locally minimal groups

W. Xi, D. Dikranjan, M. Shlossberg and D. Toller

Shamoon College of Engineering, Israel
Email: menacsh@sce.ac.il

We study locally compact groups having all dense subgroups (locally) minimal. We call such groups *densely (locally) minimal*. In 1972 Prodanov proved that the infinite compact abelian groups having all subgroups minimal are precisely the groups $\mathbb{Z}_p$ of p-adic integers. In [2], we extended Prodanov's theorem to the non-abelian case at several levels. In this paper, we focus on the densely (locally) minimal abelian groups.

We prove that in case that a topological abelian group G is either compact or connected locally compact, then G is densely locally minimal if and only if G either is a Lie group or has an open subgroup isomorphic to $\mathbb{Z}_p$ for some prime p. This should be compared with the main result of [1]. Our Theorem C provides another extension of Prodanov's theorem: an infinite locally compact group is densely minimal if and only if it is isomorphic to $\mathbb{Z}_p$. In contrast, we show that there exists a densely minimal, compact, two-step nilpotent group that neither is a Lie group nor it has an open subgroup isomorphic to $\mathbb{Z}_p$.

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The complexity of cojugacy equivalence relation of interval dynamical systems

Benjamin Vejnar and Henk Bruin

Charles University, Czech Republic
Email: vejnar@karlin.mff.cuni.cz

We study the complexity of the conjugacy equivalence relation between dynamical systems (DS) using the methods of invariant descriptive set theory. In this context, equivalence relations on Polish spaces are compared one to each other by Borel reductions. We provide an overview of known results dealing with (minimal) Cantor DS, interval invertible DS or shifts with specification. Additionally, we sketch a proof that interval DS can be classified by countable structures. This result confirms a special case of Hjorth's conjecture claiming that every equivalence relation induced by a continuous action of the homeomorphism group of $[0, 1]$ on a Polish space is classifiable by countable structures.

2010 MSC: Primary 37E05; Secondary 54H20.

Sensitive semigroup actions on continua

Suhua Wang

Jiangsu University of Science and Technology, China

In this talk, we will introduce some work about the existence of sensitive semigroup actions on continua.

Algebraic entropy on topologically quasihamiltonian groups

Wenfei Xi, Menachem Shlossberg and Daniele Toller

Nanjing University of Finance & Economics, China

We study the algebraic entropy of continuous endomorphisms of compactly covered, locally compact, topologically quasihamiltonian groups. We provide a Limit-free formula which helps us to simplify the computations of this entropy. Moreover, several Addition Theorems are given. In particular, we prove that the Addition Theorem holds for every group endomorphism of aquasihamiltonian torsion FC-group (e.g., a Hamiltonian group).

Topological conjugation classes of tightly transitive subgroups of Homeo$_+(\mathbb{S}^1)$

Hui Xu

Soochow University, China

Let Homeo$_+(\mathbb{S}^1)$ denote the group of orientation preserving homeomorphisms of the circle $\mathbb{S}^1$. A subgroup G of Homeo$_+(\mathbb{S}^1)$ is tightly transitive if it is topologically transitive and no subgroup H of G with $[G : H] = \infty$ has this property; is almost minimal if it has at most countably many nontransitive points. In the paper, we determine all the topological conjugation classes of tightly transitive and almost minimal subgroups of Homeo$_+(\mathbb{S}^1)$ which are isomorphic to $\mathbb{Z}^n$ for any integer $n \geq 2$.