# Complex Geometry IMT-SCU

October 15-21, 2018, Chengdu, China

**Motivation**

We are starting a cooperation between the Institut de Mathématiques de Toulouse (IMT, Toulouse, France) and the Mathematics Department of Sichuan University (SCU, Chengdu, China).

The first joint event is planned in Chengdu. It consists of a school (two mini-courses), followed by a workshop (9 lectures), thematically oriented towards complex geometry.

**Scientific Committee**

- Jean-Pierre Demailly (French Academy of Sciences)
- Vincent Guedj (IMT)
- An-Min Li (Sichuan University)
- Xiangyu Zhou (Chinese Academy of Sciences)

**Organizing Committee**

- Bohui Chen (Sichuan University)
- Xiaojun Chen (Sichuan University)
- Yuxin Ge (IMT)
- Li Sheng (Sichuan University)

**Financial support**

- ANR GRACK project
- Institut de Mathématiques de Toulouse
- Sichuan University
- Tianyuan Fund for Mathematics

# Minicourses (October 15-19, 2018)

## I. Singular Kähler-Einstein metrics (by A. Zeriahi)

The goal of this course is to outline the proof of the solution of the Calabi conjecture by S.-T.Yau. This implies in particular the existence of a unique Kähler-Ricci flat metric in any given Kähler class on a compact Kähler manifold with trivial first Chern class.

We will also explain how Pluripotential theory allows to solve degenerate complex Monge-Ampère equations. A consequence is the extension of Yau's theorem and the existence of a singular Kähler-Ricci flat metric on any projective variety with mild singularities and trivial first Chern class.

*References:*

1. S. T. Yau: On the Ricci curvature of a compact Kähler manifold and the complex Monge-Ampère equation. CPAM 31 (3) (1978), 139-411.

2. P. Eyssidieux, V. Guedj, A. Zeriahi: Singular Kähler-Einstein metrics. JAMS 22 (2009), no 3, 607-639.

3. V. Guedj, A. Zeriahi: Degenerate complex Monge-Ampère equations. EMS Tracts in Mathematics, V. 26, 2017.

## II. Varieties with trivial first Chern class (by H. Guenancia)

The goal of this course is to explain the proof of a decomposition theorem due to Beauville and Bogomolov which asserts that any compact Kähler manifold with trivial first Chern class admits a finite unramified cover which splits as a product of a torus, Calabi-Yau varieties and irreducible holomorphic symplectic varieties. One of the essential tools of the proof is the existence of a Kähler Ricci-flat metric, guaranteed by Yau's theorem.

Eventually, we will move on to the case of projective varieties with mild singularities and trivial first Chern class, explaining the very recent generalization by Höring and Peternell. Here again, the existence of singular Kähler-Ricci flat metrics is crucial. The proof is much more lengthy, and we will only provide a sketch of it.

*References:*

1. A. Beauville, Variétés dont la première classe de Chern est nulle, J. Differential Geom. 18 (1983), no. 4, 755-782.

2. A. Höring and T. Peternell, Algebraic integrability of foliations with numerically trivial canonical bundle, ArXiv:1710.06183.

3. D. Greb, H. Guenancia, S. Kebekus, Klt varieties with trivial canonical class - Holonomy, differential forms, and fundamental groups, ArXiv:170401408.

## Schedule

|  | Monday 15th | Tuesday 16th | Wednesday 17th | Thursday 18th | Friday 19th |
|---|---|---|---|---|---|
| 9:00-10:00 | Zeriahi | Zeriahi | Zeriahi | Zeriahi | Zeriahi |
| 10:30-11:30 | Zeriahi | Zeriahi | Guenancia | Zeriahi | Zeriahi |
| 14:00-15:00 | Guenancia | Guenancia |  | Guenancia | Guenancia |
| 15:30-16:30 | Guenancia | Guenancia |  | Guenancia | Guenancia |

# Workshop (October 20-21, 2018)

## Speakers

- Bohui Chen (Sichuan University)
- Xiaojun Chen (Sichuan University)
- Huijun Fan (Peking University)
- Yuxin Ge (IMT)
- Eveline Legendre (IMT)
- Chinh H. Lu (University of Paris 11)
- Dan Popovici (IMT)
- Zhenlei Zhang (Capital Normal University)
- Langfeng Zhu (Wuhan University)

## Schedule

### Saturday October 20

| | | |
|---|---|---|
| 9:00 - 10:00 | Lu, Chinh H. | University of Paris 11 |
| | Complex Monge-Ampère equations with prescribed singularity | |
| 10:30 - 11:30 | Fan, Huijun | Peking University |
| | Recent progress in cohomological field theories and mirror symmetry | |
| 14:00 - 15:00 | Chen, Bohui | Sichuan University |
| | Quantum Kirwan Morphisms for Abelian case | |
| 15:30 - 16:30 | Popovici, Dan | IMT |
| | Adiabatic Limit in the Context of Complex Structures | |
| 17:00 - 18:00 | Zhu, Langfeng | Wuhan University |
| | Siu's lemma, optimal $L^2$ extension and applications | |

### Sunday October 21

| | | |
|---|---|---|
| 9:00 - 10:00 | Zhang, Zhenlei | Capital Normal University |
| | Congerence of Kahler-Ricci flow | |
| 10:00 - 10:10 | Photo | |
| 10:30 - 11:30 | Chen, Xiaojun | Sichuan University |
| | Calabi-Yau algebras and the shifted noncommutative symplectic structure | |
| 14:00 - 15:00 | Legendre, Eveline | IMT |
| | The Einstein-Hilbert functional in Sasaki geometry | |
| 15:30 - 16:30 | Ge, Yuxin | IMT |
| | Conformally compact Einstein manifolds in dimension 4 | |

## Titles and Abstracts

**Chen, Bohui (Sichuan University)**

**Title: Quantum Kirwan Morphisms for Abelian case**

**Abstract:** Let $X$ be a symplectic manifold with a compact Lie group $K$ Hamiltonian action and $Z$ be a symplectic reduction. Kirwan shows that the morphism from equivariant cohomology ring of $X$ to cohomology ring of $Z$ is surjective. We introduce augmented symplectic vortex to quantize the morphisms when $K$ abelian.

**Chen, Xiaojun (Sichuan University)**

**Title: Calabi-Yau algebras and the shifted noncommutative symplectic structure**

**Abstract:** The notion of Calabi-Yau algebras was introduced by Ginzburg in 2007 and has widely been studied since then. In this talk, we show that for a Koszul Calabi-Yau algebra, there is a shifted bi-symplectic structure on the cobar construction of its co-unitalized Koszul dual coalgebra, and hence its DG representation schemes have a shifted symplectic structure, in the sense of Pantev et. al. Based on a joint work with F. Eshmatov.

**Fan, Huijun (Peking University)**

**Title: Recent progress in cohomological field theories and mirror symmetry**

**Abstract:** As the first part of the talk, I will recall the origin of the mirror symmetry phenomenon and then turn to the earlier explanation and conjectures of the mirror symmetry in mathematics. Then I will report the recent exciting progress in quantum singularity (FJRW) theory, LG/CY correspondence, construction of the GLSM (gauged linear sigma model) invariants, and the computation of these quantum invariants.

**Ge, Yuxin (IMT)**

**Title: Conformally compact Einstein manifolds in dimension 4**

**Abstract:** We show some compactness result of 4-dimensional conformally compact Einstein manifolds under the suitable assumptions on the topology of the manifolds, on the compactness of their conformal infinity and on some suitable conformal invariants. We discuss also the existence and uniqueness of such manifolds when the conformal infinity is close to the standard 3-sphere. These are joint works with Alice Chang and with Alice Chang and Jie Qing.

**Legendre, Eveline (IMT)**

**Title: The Einstein-Hilbert functional in Sasaki geometry**

**Abstract:** I will explain conical Kähler space/Sasaki manifolds are and open questions on existence of constant scalar curvature metrics on these spaces. I will introduce the Einstein-Hilbert functional as an obstruction and discuss results and open questions about it.

**Lu, Chinh H. (University of Paris 11)**

**Title: Complex Monge-Ampère equations with prescribed singularity**

**Abstract:** Let $X$ be a compact Kähler manifold of complex dimension $n$. We develop a pluripotential theory for a class of quasi-plurisubharmonic functions outside the energy class $E$. Using a uniform estimate a la Kolodziej we solve complex Monge-Ampère equation within an arbitrary singularity class. As a consequence, we fully confirm a conjecture of Boucksom-Eyssidieux-Guedj-Zeriahi on log concavity of the volume function. This is joint work with Tamas Darvas and Eleonora Di Nezza.

**Popovici, Dan (IMT)**

**Title: Adiabatic Limit in the Context of Complex Structures**

**Abstract:** We adapt to the case of a complex structure on a given manifold the well-known adiabatic limit construction for Riemannian foliations. Specifically, for any complex constant $h$, we deform the Poincaré differential $d = \partial + \bar{\partial}$ to the new differential operator $d_h = h\partial + \bar{\partial}$ which gives rise to a cohomology that is canonically isomorphic to the De Rham cohomology except when $h = 0$ and to two Laplace-type second-order elliptic differential operators that induce a Hodge theory for this cohomology. We will then present our main result, a characterisation of the dimensions of the vector spaces featuring in the Frölicher spectral sequence of the given compact complex manifold in terms of the number of small eigenvalues of these Laplacians and their asymptotics. If time permits, we will also present applications to the deformation theory of complex structures.

**Zhang, Zhenlei (Capital Normal University)**

**Title: Congerence of Kahler-Ricci flow**

**Abstract:** In the talk I will survey the results of convergence of Kahler-Ricci flow. I will focus on the infinite time case. The results partially confirm the Analytic Minimal Model Program.

**Zhu, Langfeng (Wuhan University)**

**Title: Siu's lemma, optimal $L^2$ extension and applications**

**Abstract:** In this talk, we will discuss an optimal $L^2$ extension theorem for holomorphic sections of the canonical bundle twisted with a singular Hermitian line bundle on weakly pseudoconvex Kähler manifolds, and the pseudoeffectivity of twisted relative pluricanonical bundles for Kähler fibrations. In our proofs, a crucial step is to find and prove a generalization of Siu's lemma, and Guan-Zhou's solution of Demailly's strong openness conjecture also plays an important role. These are joint works with Professor Xiangyu Zhou.

*All talks will take place at Room W303 of School of Mathematics, Sichuan University.