# Noncommutative Geometry and Representation Theory

Tianyuan Mathematical Institute in Southwest China
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## Abstracts

**Kuok Fai Chao** (Shanghai University)
*On the Langlands programme and noncommutative geometry.*

In this talk, we would like to introduce how we can connect Langlands programme with noncommutative geometry. More precisely, we will use the dual space of Lie groups as the main object to construct the link between local Langlands correspondences and the reduced $C^*$-algebras. This is a joint work with Hang Wang and King-Fai Lai.

**Heath Emerson** (University of Victoria)
*Dirac classes, duality, and the irrational rotation algebra.*

We discuss a method of using $KK$-theory to study the noncommutative geometry of crossed-products of discrete groups acting smoothly on smooth manifolds. The main idea is to focus on the 'Dirac class' of such an action. We will mainly focus on some low-dimensional examples, especially the irrational rotation action of the integers on the circle, discuss Dirac classes in this context, show how to compute their $K$-theory pairing topologically, and give a new and geometric proof of Connes' Poincaré duality result for the irrational rotation algebra, using transversality ideas.

**Yoshiyasu Fukumoto** (East China Normal University)
*$G$-homotopy invariance of analytic signatures of proper $G$-manifolds.*

In this talk we will discuss the Strong Novikov conjecture (SNC), the Novikov conjecture (NC), and the extension to proper actions of locally compact groups. More specifically, NC is deduced from SNC by the homotopy invariance of $G$-index of the signature operator in the $K$-theory of group $C^*$-algebra. This homotopy invariance is proved using the Hilsum and Skandalis' deformation argument.

**Fei Han** (National University of Singapore)
*$T$-duality in an $H$-flux and exotic twisted equivariant cohomology theories.*

$T$-duality is an equivalence of two physical theories (quantum field theories or string theories) with different spacetime geometries. It originates from string theories, finds applications in condensed matter physics and has profound connections with topology/geometry. In this talk, I will first briefly review the elegant mathematical formulation of $T$-duality in an $H$-flux by Bouwknegt-Evslin-Mathai. Then I will describe our joint work with Mathai about developing exotic twisted equivariant cohomology theories in order to enhance the $T$-duality by capturing the exchange of winding and momentum, a fundamental feature of $T$-duality.

**Nigel Higson** (Penn State University)
*Another look at discrete series representations from the point of view of the Dirac operator.*

There are long-established links between Harish-Chandras parametrization of the discrete series representations, the Dirac operator and index theory that were pioneered by Atiyah and Schmid, Connes and Kasparov, and Lafforgue. In this talk I will try to examine discrete series and the Dirac operator from a different point of view, which is closer to Harish-Chandra's original work using orbital integrals,

but which is nonetheless grounded in $C^*$-algebras and noncommutative geometry. This is joint work with Tsuyoshi Kato.

**Tsuyoshi Kato** (Kyoto University)
*Higher Nahm transform in noncommutative geometry.*

Anti-self-dual (ASD) connections over a compact four manifold X attain the critical values on Yang-Mill functional. Nahm transform is a correspondence from a vector bundle with a connection on X to another vector bundle with a connection on the Picard torus. In the case of four torus, it transforms ASD to ASD. In this talk we propose a noncommutative geometric version of Nahm transform, which generalizes Connes-Yang-Mills functional via higher Dixmier traces. This is joint work with Hirofumi Sasahira and Hang Wang.

**Yasuyuki Kawahigashi** (University of Tokyo)
*Conformal field theory and operator algebras.*

I will present recent progress of operator algebraic studies of chiral conformal field theory. I will emphasis representation theoretic aspects related to modular tensor categories and interactions with theory of vertex operator algebras.

**Yosuke Kubota** (RIKEN)
*Reconstructing the Bost–Connes semigroup actions from $K$-theory.*

In this talk, we introduce our recent result on the complete classification of the Bost–Connes systems and their underlying C*-algebras. The Bost-Connes system is a $C^*$-dynamical system attached with a number field, which realizes the arithmetics of the number field through its dynamics. The underlying $C^*$-algebra is defined as the crossed product $C^*$-algebra of a semigroup action defined by using the Artin reciprocity map in class field theory. We show that the Bost-Connes semigroup action attached to two number fields are isomorphic if and only if their crossed product $C^*$-algebras have the same $K$-theoretic data. Together with reconstruction results in number theory by Cornelissen–de Smit–Li–Marcolli–Smit, we conclude that two Bost–Connes $C^*$-algebras are isomorphic if and only if the original number fields are isomorphic. This is a joint work with Takuya Takeishi.

**Yang Liu** (Max Planck Institute)
*Modular curvature and hypergeometric functions.*

In noncommutative geometry, an essential question is to extend the notion of metric and curvature in Riemannian geometry to noncommutative spaces in a operator theoretical framework. A fundamental feature, in contrast to Riemannian geometry, is the fact that metrics are parametrized by noncommutative coordinates. In the conformal geometry of noncommutative tori, the new structure in the modular analog of the Gaussian curvature consists of two spectral functions, which compress the Ansatz caused by the noncommutativity between the metric coordinate and its derivatives. In the first part of the talk, I will explain the higher dimensional generalization of a fantastic functional equation between them due to Connes and Moscovici. In the second part, I will show that hypergeometric functions are the build blocks of those spectral functions. A surprising discovery, obtained by combining the power of hypergeometric functions and computer algebra systems, is that Connes-Moscovici functional relation can be extended to a continuous family with respect to the dimension parameter.

**Hitoshi Moriyoshi** (Nagoya University)
*Fuzzy sphere and the Ginsparg-Wilson index.*

In the lattice gauge theory there has been investigations to approximate the Dirac operator by finite dimensional objects induced from the lattice. Fuzzy sphere, the endomorphism ring on a finite-dimensional irreducible $SO(3)$-space, can be considered as one of such attempts. However, the Fredholm index of an operator is uniquely determined in the finite dimensional case, namely the difference of dimensions. In order to remedy the defect, Ginsparg and Wilson introduced a new relation for such operators to get nontrivial index. In this talk I shall explain about the universal Ginsparg-Wilson algebra and study the $K$-theory. When we apply it for the Cayley transform, we can recover the ordinary index for Dirac operators. Also an explicit construction of such operator on Fuzzy sphere is mentioned. This is a joint work with Toshi Natsume.

**Toshikazu Natsume** (Ritsumeikan University)
*Ginsparg-Wilson operators and a duality.*

In lattice gauge theory mathematical physicists introduced a notion of Ginsparg-Wilson operators. These operators are defined on Hilbert spaces with symmetry given as alternatives of odd operators. Those operators are analyzed from mathematical, particularly $K$-theoretic viewpoint. This is partly joint work with Hitoshi Moriyoshi.

**Ryszard Nest** (University of Copenhagen)
*Determinants and higher group cocycles.*

We describe the construction of a universal three cocycle, the higher dimensional analogue of the universal central extension of the loop groups appearing in string theory. The construction also provides an explicit semi-universal formula for the values of the multiplicative Chern character on the third algebraic $K$group, generalising the Tate symbol on the $K$theory of fields. In analogy with the two cocycle associated to a polarized Hilbert space, the construction extends the known formulas for Tate spaces to the case of smooth, instead of analytic, functions and to the group of invertible elements in non-commutative algebras.

**Yanli Song** (Washington University in St Louis)
*Orbital integral and group $C^*$-algebra.*

In this case, I will discuss the $K$-theory of group $C^*$-algebra and Connes-Kasparov isomorphism for reductive Lie group. The main method we used is the orbital integral introduced by Harish-Chandra. I will try to explain some connections between the representation theory of Lie group and indices of Dirac operators. This is a joint work with Nigel Higson and Xiang Tang.

**Fedor Sukochev** (University of New South Wales)
*Connes integration formula for the noncommutative plane.*

Our aim is to prove the integration formula on the noncommutative (Moyal) plane in terms of singular traces similar to the formula featuring in Connes Trace Theorem for compact Riemannian manifolds. Joint work with Dmitriy Zanin.

**Doman Takata** (University of Tokyo)
*An analytic index theory for infinite-dimensional manifolds and $KK$-theory.*

The Atiyah-Singer index theorem is a monumental work in geometry and topology. The overall goal of my research is to formulate and prove an infinite-dimensional version of this theorem. For this purpose, it is natural to begin with simple cases, and my current problem is the following: For infinite-dimensional manifolds equipped with a "proper and cocompact" action of the loop group of the circle, construct a loop group equivariant index theory, from the viewpoint of $KK$-theory.

Although this project has not been completed, I have constructed several core objects for the analytic side of this problem, including a Hilbert space regarded as an "$L^2$-space", in `arXiv:1701.06055` and `arXiv:1709.06205`. In this talk, I am going to report on the progress made so far.

**Guo Chuan Thiang** (University of Adelaide)
*Crystallographic $T$-duality and the bulk-boundary correspondence principle.*

The classical Gohberg-Krein index theorem for Toeplitz operators can be understood physically as a bulk-boundary correspondence for supersymmetric spectrally-gapped Hamiltonians on a 1D chain, in which a "bulk winding number" equals the number of "boundary zero modes" at a chain termination. I will explain how new index theorems may be derived by extending the heuristic to the crystallographic setting. For the $pg$-wallpaper group, this involves subtle mod-2 indices in twisted $K$-theory, which are computable though Baum-Connes isomorphisms. A glimpse of a whole zoo of "crystallographic $T$-dualities" will also be given.

**Qin Wang** (East China Normal University)
*Persistence approximation property for maximal Roe algebras.*

The persistent approximation property for quantitative K-theory of filtered C?-algebras was introduced by H. Oyono-Oyono and G. Yu. In this talk, we shall discuss the persistence approximation property for maximal Roe algebras of coarse spaces, and its applications to the maximal coarse Baum-Connes conjecture. This is joint work with Zhen Wang.

**Xianjin Wang** (Chongqing University)
*Coarse embedding and the equivariant coarse Baum-Connes conjecture.*

Let $G$ be a countable discrete group and $X$ a bounded geometry metric space with a proper and isometric $G$-action. The equivariant coarse Baum-Connes conjecture asserts that the equivariant higher index map $\mathrm{ind}^G : \lim_{d\to\infty} K_*^G(P_d(X)) \to K_*(C^*(X)^G)$ is an isomorphism, where $K_*^G(P_d(X))$ is the $G$-equivariant $K$was -homology of the Rip complex $P_d(X)$, and $K_*(C^*(X)^G)$ is the $K$-theory of the equivariant Roe algebra $C^*(X)^G$. In this talk, I will introduce our resent works on the equivariant coarse Baum-Connes conjecture provided that $X$ or $X/G$ and $G$ are coarsely embeddable into Hilbert spaces.

**Shilin Yu** (Texas A&M University)
*Orbit method from a new perspective.*

Kirillov's coadjoint orbit method suggests that irreducible unitary representations can be constructed as geometric quantization of coadjoint orbits of the group. Except for a lot of evidence, the quantization scheme meets strong resistance in the case of noncompact reductive groups. I will give a new perspective on the problem using deformation quantization of symplectic varieties and their Lagrangian subvarieties. This is joint work in progress with Conan Leung.

**Dmitriy Zanin** (University of New South Wales)
*Connes Character Formula for locally compact spectral triples.*

We select a natural (and large) class of locally compact spectral triples. For example, all Riemannian manifolds belong to these class, and so do noncommutative Euclidean spaces. For every spectral triple in this subclass, we demonstrate the validity of all 3 variants of Connes Character Formula:

(1) As asymptotic of heat semigroup.
(2) As analytic continuation of $\zeta-$function to a bigger half-plane.
(3) As an equality in terms of singular traces.

**Dapeng Zhou** (East China Normal University)

*Higher index theory and quantitative operator $K$-theory.*

Higher index theory is closely related to a range of mathematical issues including topology of manifold and metrics of positive scalar curvature. The Baum-Connes conjecture and the coarse Baum-Connes conjecture are algorithms to compute the higher indices of elliptic differential operators. The quantitative operator $K$-theory, a refined version of classical operator $K$-theory, is a powerful tool to study these conjectures. In this talk, I will start with the basic definitions and facts from higher index theory, and I will give a survey on the recent development of Baum-Connes conjecture and controlled $K$-theory method.