2021 四川大学代数与数论在线论坛

2021 年 12 月 11 日 — 2021 年 12 月 13 日

A new total positivity on the flag manifolds

何旭华 (香港中文大学)

摘要

The totally nonnegative flag manifold was introduced by Lusztig. It has enriched combinatorial, geometric, and Lie-theoretic structures. In this talk, we introduce the J-total positivity on the full flag manifold of an arbitrary Kac-Moody group. This new total positivity is compatible with the decomposition of the full flag into the J-Richardson varieties ${}^{J}\mathcal{B}_{w_1,w_2}$. Moreover, the J-total positivity on the full flag provides a model for the (ordinary) totally nonnegative partial flag manifold. We discovered that the J-total positivity admits the product structure ${}^{J}\mathcal{B}_{w_1,w_2,>0} \cong {}^{J}\mathcal{B}_{w_1,w_3,>0} \times {}^{J}\mathcal{B}_{w_3,w_2,>0}$ as stratified spaces. Such product structure plays a crucial in the understanding of various geometric structures on the total positivity. As an application, we prove that the closure of each (ordinary) totally positive Richardson variety is homeomorphic to a ball, confirming a conjecture of Galashin, Karp and Lam.

K-stability and Fujita approximation

李驰 (罗格斯大学)

摘要

I will explain a conjecture about the first Riemann-Roch coefficients of big line bundles over projective varieties. This conjecture involves the concept of restricted volumes and strengthens the Fujita approximation theorem on volumes of big line bundles. Then I will explain how this conjecture arises in the non-Archimedean aspect of the Yau-Tian-Donaldson conjecture, which is about the relation between K-stability of projective manifolds and the existence of canonical Kahler metrics.

Slopes of modular forms and the ghost conjecture of Bergdall-Pollack (1)

刘若川 (北京大学)

摘要

In classical theory, slopes of modular forms are p-adic valuations of the eigenvalues of the U_p-operator. On the Galois side, they correspond to the p-adic valuations of eigenvalues of the crystalline Frobenius on the Kisin's crystabelian deformations space. I will report on a joint work in progress in which we proved a version of the ghost conjecture of Bergdall and Pollack. This has many consequences in the classical theory, such as some cases of Gouvea-Mazur conjecture, and some hope towards understanding irreducible components of eigencurves. On the Galois side, our theorem can be used to prove certain integrality statement on slopes of crystalline Frobenius on Kisin's deformation space, as conjectured by Breuil-Buzzard-Emerton. This is a joint work with Nha Truong, Liang Xiao and Bin Zhao. My talk will be followed by Liang Xiao's talk.

Finite generation, K-stability, and canonical metrics

刘雨晨 (西北大学)

摘要

K-stability is an algebraic stability condition characterizing the existence of canonical metrics on algebraic varieties. The famous Yau-Tian-Donaldson (YTD) conjecture predicts that a Fano variety admits a Kähler-Einstein metric if and only if it is K-polystable. In this talk, I will introduce a finite generation problem in K-stability that is closely related to the YTD conjecture, and discuss a solution to this problem using birational geometry. As a consequence, we confirm the YTD conjecture for all (possibly singular) Fano varieties combining with the variational approach. This is based on joint work with Chenyang Xu and Ziquan Zhuang.

On the center of small quantum groups

单芃 (清华大学)

摘要

We will explain a geometrical realisation of the center of small quantum groups in terms of cohomology of some affine Spaltenstein varieties. As application, this gives some evidence for a conjecture of Lachowska and Qi on the dimension of the center of small quantum groups. This is based on a joint work with R. Bezrukavnikov, P. Boixeda Alvarez and E. Vasserot.

Moduli of Higgs bundles and enumerative geometry

沈俊亮 (耶鲁大学)

摘要

Moduli of Higgs bundles and Hitchin's integrable systems play important roles in many branches of modern mathematics. In this talk, we will discuss the interaction between Higgs moduli spaces and enumerative geometry of Calabi-Yau 3-folds, where the latter is an active area motivated by string theory and mirror symmetry. We will explain how such connections lead to progress to both questions originated in enumerative geometry and conjectures concerning geometry of the Higgs moduli spaces. We will particularly focus on the role played by Ngo's support throrem, which is an important ingredient in his proof of the fundamental lemma of the Langlands program.

Achimedean period relations and automorphic period relations

孙斌勇 (浙江大学)

摘要

It was known to Euler that $\zeta(2k)$ is a rational multiple of $\pi \cdot 2k$, where $\zeta(\cdot)$ is the Euler-Riemann zeta function, and k is a positive integer. Following the pioneering works of G. Shimura, P. Deligne and etc., D. Blasius proposed a conjecture which asserts that similar rationality results hold for very general automorphic L-functions. We confirm Blasius's conjecture in two cases: the standard L-functions of symplectic type (joint with Dihua Jiang and Fangyang Tian), and the Rankin-Selberg L-functions for $\mathrm{GL}(n) \times \mathrm{GL}(n-1)$ (joint with Jian-Shu Li and Dongwen Liu). The key ingredient is the Archimedean period relations for the modular symbols at infinity. These two cases have already been studied by many authors, including Harris-Lin, Grobner-Raghuram, Harder-Raghuram, Januszewski, Grobner-Lin, etc.

Slopes of modular forms and the ghost conjecture of Bergdall-Pollack (2)

肖梁 (北京大学)

摘要

This is a continuation of that of Ruochuan Liu's talk.

Cohomology sheaves of stacks of shtukas

薛聪 (巴黎大学)

摘要

The cohomology of stacks of shtukas plays an important role in the Langlands correspondence for function fields. In this talk, we will recall the definition of the cohomology sheaves of stacks of shtukas. We will explain the finiteness and smoothness properties of the cohomology sheaves. We will also talk about some applications of these properties.

Character sheaves for graded Lie algebras

薛婷 (墨尔本大学)

摘要

The Springer theory for reductive algebraic groups plays an important role in representation theory. It relates nilpotent orbits to irreducible representations of Coxeter groups. We discuss a Springer theory for graded Lie algebras and describe the character sheaves arising in this setting. Via a nearby cycle construction irreducible representations of Hecke algebras of complex reflection groups at roots of unity enter the description of character sheaves. Recent work of Lusztig and Yun relates character sheaves to irreducible representations of trigonometric double affine Hecke algebras. We will discuss the connection between the work of Lusztig-Yun and our work, and some conjectures arising from this connection, if time permits. This is based on joint work with Kari Vilonen and partly with Misha Grinberg.

Survey on the Bogomolov conjectures

袁新意 (北京大学)

摘要

This talk will be a summary of various versions of the Bogomolov conjecture, including the original Bogomolov conjecture, the geometric Bogomolov conjecture, the uniform Bogomolov conjecture, and the relative Bogomolov conjecture. All but the last one have been proved by the works of many people in the past three decades.