

Mini workshop on Positivity

Tianyuan mathematical center in Southwest China
25-27 June 2024

Fundamental questions about regular operators between Banach lattices

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Abstract

Many old questions concerning regular operators between Banach lattices have received little attention in recent years, even though they remain not being completely answered. In many cases, even what has been established seems not to be as well known as it ought to be. In this talk I will, time permitting, look at four such questions, describe what is known and, more importantly, point out significant gaps in our knowledge that are worthy of further investigation. The four topics are:

1. For which pairs of Banach lattices X and Y are all bounded linear operators from X into Y regular?
2. When can lattice operations be calculated using the Riesz-Kantorovich formulae?
3. When is it true that $|T^*| = |T|^*$?
4. When do spaces of regular operators, under the regular norm, form (isometrically or isomorphically) a classical Banach lattice?

Free and universal objects in categories of vector lattices

Marcel de Jeu
Leiden University and University of Pretoria

Abstract

A classical result in universal algebra on 'equational classes' implies that, for example, free vector lattices over sets exist, as do free vector lattice algebras over vector spaces. In this expository lecture, I shall explain why this is the case and how this can be extended to an existence theorem for free objects in what could be called 'implicational classes', where equational identities between elements are implied by the validity of other (possibly none) equational identities. This yields the existence of, for example, free rings over sets with the property that $ab + c + d = 0$ whenever $ac - d = b$, and also, of greater practical relevance, free f -algebras over sets.

No particular knowledge is necessary to understand the contents of this lecture, which is on basic algebraic principles that are hardly ever part of the curriculum.

Tensor products of vector and Banach lattices

Vladimir Troitsky
University of Alberta

Abstract

In the talk, we will provide an overview of several tensor products for the categories of vector and Banach lattices. We will present a simplified construction of the vector lattice tensor product (originally due to Fremlin), as well as its properties. We will discuss the positive versions of the projective and the injective tensor products.

\mathbb{L} -functional analysis

Marten Wortel
University of Pretoria

Abstract

Inspired by recent work in ergodic theory by Edeko, Haase, and Kreidler that goes back to Kaplansky's 1953 paper on Kaplansky-Hilbert modules, as well as the theory of stochastic processes in Riesz spaces pioneered in South Africa since the 2000's, we generalize functional analysis by replacing the scalars (\mathbb{R} or \mathbb{C}) by a (real or complex) Dedekind complete unital f -algebra \mathbb{L} . I will start by explaining why such a theory is relevant and interesting, and then I will explain some of our results on \mathbb{L} -Banach and \mathbb{L} -Hilbert spaces and show how their proofs compare with the classical case.

This is joint work with Walt van Amstel, Eder Kikianty, Miek Messerschmidt, Luan Naude, Chris Schwanke, Jan Harm van der Walt (all from Pretoria), and Mark Roelands (Leiden).

Some open problems on locally solid convergences

Eugene Bilokopytov
University of Alberta

Abstract

In this talk we will survey the recent work by Conradie, Troitsky, van der Walt and the speaker on locally solid convergences with the focus on the questions which were left open. The theory of locally solid convergences generalizes the locally solid topologies but also includes other classical notions of convergences on a vector lattice including order and relative uniform convergences. The questions are either of the topological nature (when does a certain convergence have a certain topological property), or about the way in which the convergence interacts with linear or lattice structure.