

成都国际数论研讨会

报告题目与摘要

(2024年6月21日至24日，成都)

1. A conjecture of Erdős on $p + 2^k$

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Abstract

Let \mathcal{U} be the set of positive odd integers that cannot be represented as the sum of a prime and a power of two. Erdős conjectured that the set \mathcal{U} is the union of an infinite arithmetic progression of positive odd integers and a set of asymptotic density zero. In this talk, we answer the above Erdős' conjecture negatively by proving the following stronger result: \mathcal{U} is not a union of finitely many infinite arithmetic progressions and a set of asymptotic density zero.

2. On generalized Sidon sets

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Abstract

A set A of integers is defined as a Sidon set if all the sums $a + a'$, $a \leq a'$, $a, a' \in A$ are distinct. In 2022, Nathanson further considered Sidon sets for linear forms. In this talk, we will present our recent results on generalized Sidon sets.

3. The exotic inverted Kloosterman sum

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Abstract

Let B be a product of finitely many finite fields containing \mathbb{F}_q , and let $\chi : B^* \rightarrow \mathbb{C}^*$ be a multiplicative character. Katz introduced the so-called exotic inverted Kloosterman sum

$$\sum_{\substack{x \in B^*, \\ \text{Tr}_{B/\mathbb{F}_q}(x) \neq 0, \text{N}_{B/\mathbb{F}_q}(x) = a}} \chi(x) e^{\frac{2\pi i}{p} \left(\frac{1}{\text{Tr}_{B/\mathbb{F}_q}(x)} \right)} \quad (a \in \mathbb{F}_q^*).$$

We explain how to use algebraic geometry to study such a sum. This is a joint work with Daqing Wan.

4. On automorphic analogues of the Bombieri-Vinogradov theorem

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Abstract

We shall present some unconditional results of Bombieri-Vinogradov type for the automorphic cases. Moreover, for a fixed residue class and a weak upper bound, the levels of distributions for the above cases can go beyond the $x^{1/2}$ -barrier, which are sufficient for some applications.

5. The reciprocity relations associated with degenerate unipoly-Dedekind DC sums

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Abstract

As the new generalization of the two types of unipoly-Dedekind type DC sums , the two types of degenerate unipoly-Dedekind type DC sums are introduced, which are obtained from the unipoly-Dedekind type DC sums by replacing type 2 unipoly-Euler functions by type 2 degenerate unipoly-Euler functions and unipoly-Euler functions by degenerate unipoly-Euler functions. By using the definitions of unipoly functions, degenerate Stirling numbers of first kind and second kind, degenerate Genocchi polynomials and numbers and other polynomials and numbers, several combinatorial identities and properties of type 2 degenerate unipoly-Euler functions and degenerate unipoly-Euler functions are obtained. The two types of degenerate unipoly-Dedekind DC sums are shown to satisfy reciprocity relations. This is a joint work with Lingling Luo and Yuankui Ma.

6. Introduction to Newton Polygons of L-functions

李吉有

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Abstract

In this talk, we first give a brief introduction to the theory of Newton polygons and Hodge polygons of L-functions associated with Laurent polynomials. Then we demonstrate through examples how to determine if a polynomial is ordinary, meaning that its Newton polygon coincides with its Hodge polygon by Wan's decomposition theorems.

7. Correlations of multiplicative functions with coefficients of automorphic L-functions

吕广世

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Abstract

In this talk, we introduce our recent work on correlations of multiplicative functions with coefficients of automorphic L-functions. In particular, nontrivial savings are achieved for shifted convolution problems on $GL(m) \times GL(2)$ ($m \geq 4$) and Hypothesis C of Iwaniec - Luo - Sarnak for the first time.

8. Isogeny-based cryptosystems and structure of isogeny graphs

欧阳毅

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Abstract

Supersingular elliptic curve Isogeny-based cryptography is one route for post-quantum cryptography. In this talk, we shall explain the mathematical problems on isogeny computation and the cryptosystems based on those problems. We'll then focus on the study of the isogeny graphs of supersingular elliptic curves and of superspecial abelian surfaces. We will explain several results about the local structure of these graphs. This is based on joint works with Zheng Xu, Songsong Li and Zijian Zhou.

9. 关于p-adic超同余式的局部-整体型定理

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Abstract

我们将介绍一个关于p-adic超同余式的局部-整体型定理，以及如何将其应用于研究截断超几何函数的算术性质。

10. Problems and results on determinants involving Legendre symbols

孙智伟

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Abstract

In this talk we focus on determinants whose entries are linear combinations of Legendre symbols. We first review the classical results in this direction, including Maxim Vsemirnov's solution of Robin Chapman's evil determinant. Then we introduce the recent developments and pose various new conjectures for further research. For example, for any prime $p > 3$ we conjecture that

$$\det \left[\left(\frac{j+k}{p} \right) + \left(\frac{j-k}{p} \right) + \left(\frac{jk}{p} \right) \right]_{1 \leq j, k \leq (p-1)/2} \\ = \begin{cases} \left(\frac{2}{p} \right) p^{(p-5)/4} & \text{if } p \equiv 1 \pmod{4}, \\ (-1)^{(h(-p)-1)/2} (1 - (2 - \frac{2}{p})h(-p)) p^{(p-3)/4} & \text{if } p \equiv 3 \pmod{4}, \end{cases}$$

where $(\frac{\cdot}{p})$ denotes the Legendre symbol, and $h(-p)$ stands for the class number of the imaginary quadratic field $\mathbb{Q}(\sqrt{-p})$.

11. 群环、半群环与零和

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Abstract

本报告介绍群环理论在零和研究方面的结果，并探讨半群环的工具在半群结构上零和幂等元堆垒问题上的应用。

12. The classification of singular Borcherds products

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Abstract

Singular Borcherds products are minimal weight modular forms on type IV symmetric domains with infinite product expansions and special zeros. Modular forms of this type are particularly interesting because they often correspond to generalized Kac-Moody algebras and vertex operator algebras. In 1995, Borcherds asked whether the number of such modular forms is finite or infinite. In this talk, we will present some recent progress on this problem.

13. Rogers-Ramanujan type identities and Nahm sums

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Abstract

Let $r \geq 1$ be a positive integer, A a real positive definite symmetric $r \times r$ matrix, B a vector of length r , and C a scalar. Nahm's problem is to describe all such A , B and C with rational entries for which

$$F_{A,B,C}(q) = \sum_{n=(n_1, \dots, n_r) \in (\mathbb{Z}_{\geq 0})^r} \frac{q^{\frac{1}{2}n^T A n + n^T B + C}}{(q)_{n_1} \cdots (q)_{n_r}}$$

is a modular form. Zagier completely solved the rank one case. When the rank $r = 2, 3$, Zagier presented many examples of (A, B, C) for which $F_{A,B,C}(q)$ appears to be a modular form. We present a number of Rogers-Ramanujan type identities involving double and triple sums, which give modular form representations for Zagier's rank two and rank three examples. We will also discuss the modularity of some other generalized Nahm sums.

14. Salem Numbers with Minimal Trace

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Abstract

A Salem number τ of degree $2d$ is a real algebraic integer greater than 1 whose other conjugates all lie in the closed disc $|z| \leq 1$, with at least one on the unit circle. The transformation $\alpha = \tau + 1/\tau + 2$ produces a totally positive algebraic integer α of degree d whose all zeros but one are in the interval $(0, 4)$. In this talk, a new method is given to optimize the lower and upper bounds for such totally positive algebraic integer α with given trace and given degree, and then the bounds for s_k . Therefore all Salem numbers of degree 32, 40 and 62 with minimal trace -3 , -4 and -6 respectively are found. This is a joint work with Qiong Chen.

15. 特征和

郝平

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Abstract

特征和基本、有趣且有用。本报告主要回顾前人关于(整数环及有限域上)特征和的若干研究工作以及我们最近几年的研究进展，主要涉及上界、均值、等分布和一些应用。

16. Counting Lattice points in central simple algebras with a given characteristic polynomial

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Abstract

Eskin, Mozes and Shah determined an asymptotic formula for integral matrices with a given irreducible characteristic polynomial over \mathbb{Z} . We'll extend this result to a central simple algebra based on our previous work about counting integral points in homogeneous spaces. This is a joint work in progress with Jiaqi Xie.

17. When is an automatic number transparent?

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Abstract

In this talk we introduce a new notion to measure the complexity of automatic numbers, which are either rational or transcendental. We study basic properties of this notion, and exhibit an algorithm to compute it. In particular, we shall characterize all the automatic numbers which are transparent. As applications, we shall also compute the complexity of some well-known automatic numbers.

18. 有限域上局部置换多项式

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Abstract

有限域上置换多项式在编码学、构造Bent函数、组合设计和有限几何等都有重要应用。本报告主要讲述我们最新定义的有限域上局部置换多项式的基本性质及其在构造置换多项式方面的应用。

19. Multiple zeta values and their variants

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Abstract

In recent years, a lot of research has been focused on the study of multiple zeta values and their variants such as multiple T-values, multiple t-values, and multiple S-values. In this talk, I will consider a common generalization of such values called multiple mixed values and describe some situations where these values can be expressed as rational linear combinations of multiple zeta values. This work is joint with Ce Xu.

20. Forms in prime variables

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Abstract

In 2010, Liu studied the quadratic forms in 10 prime variables. Inspired by this work, people started to study forms in prime variables and related topics such as the Roth type result. In this talk, we shall give a survey on this topic.

21. On the multiplicity of the eigenvalues of discrete tori

赵永强

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Abstract

It is well known that the standard flat torus $T^2 = R^2/Z^2$ has arbitrarily large Laplacian-eigenvalue multiplicities. Consider the discrete torus $C_N * C_N$ with the discrete Laplacian operator; we prove, however, that the eigenvalue multiplicities are uniformly bounded for any N , except for the eigenvalue one when N is even. In fact, similar phenomena also hold for higher-dimensional discrete tori. In this talk, we will outline a proof of the uniformly bounded multiplicity result. This is a joint work with Bing Xie and Yigeng Zhao.