2023 Summer Mini-Symposium on Celestial Mechanics and Related Algebraic, Geometric, and Topological Problems

Sichuan University

Organizer: Shiqing Zhang

July 4 and July 5, 2023

地点：长江数学中心 302

Zoom Meeting ID: 792 779 3345

会议议程

Schedule

	开幕式 Opening Ceremony	主持人 Organizer
7 月 4 日 8:50-9:00	张世清教授致欢迎辞 Welcome Remark by Dr. Shiqing Zhang	谢之福 Zhifu Xie

7 月 4 日会议报告安排 July 4, 2023			
时间 Time	报告人 Presenter	题目 Title	主持人 Organizer
9:00-9:40	Cristina Stoica	A Continuation Theorem in Classical Mechanics	祝书强 Shuqiang Zhu
9:50-10:30	Marshall Hampton	Music of the Spheres: Celestial Mechanics and FM Synthesis	邓义杨 Yiyang Deng
10:40-11:20	谢之福 Zhifu Xie	On the Uniqueness of Convex Central Configurations in the Planar 4-Body Problem	张世清 Shiqing Zhang
11:30-12:10	于翔 Xiang Yu	On the Finiteness of Four-Body Central Configurations	谢之福 Zhifu Xie

7 月 5 日会议报告安排 July 5, 2023			
时间 Time	报告人 Presenter	题目 Title	主持人 Organizer
9:00-9:40	Edward Belbruno	Applications of the McGehee Transformation to Singularities in Astrophysics, Cosmology and Celestial Mechanics	谢之福 Zhifu Xie
9:50-10:30	Ernesto Pérez-Chavela	Relative Equilibria on the Sphere	祝书强 Shuqiang Zhu
10:40-11:20	Alexander Wires	Deconstruction/Reconstruction of Extensions	张世清 Shiqing Zhang
11:30-12:10	谢之福 Zhifu Xie	Interval Analysis, Krawczyk Operator and SageMath	于翔 Xiang Yu

Time: Beijing time 9:00 am -- 9:40 am, July 4, 2023 (Toronto time: 9:00 pm – 9:40 pm, July 3th, 2023)
Speaker: Cristina Stoica
Affiliation: Wilfrid Laurier University, Canada

Title: A continuation theorem in classical mechanics

Abstract: I will discuss a general framework for defining restricted problems in mechanical systems with symmetry. A typical example is that of the restricted three body problem in celestial mechanics. I will state a theorem concerning the persistence of dynamical features from restricted to non-restricted problems. The techniques used are based on scaling, symplectic reduction and the description of the linearization operator at a relative equilibrium.

I will present some applications to systems such as a gravitationally coupled rigid body and a point mass, and the spherical double pendulum with a small mass at the free end.

Time: Beijing time 9:50 am -- 10:30am, July 4, 2023 (Minneapolis time: 8:50 pm – 9:30 pm, July 3rd, 2023)

Speaker: Marshall Hampton
Affiliation: University of Minnesota Duluth, USA

Title: Music of the Spheres: Celestial Mechanics and FM Synthesis

Abstract: How can we use math to create and control interesting sounds? This talk will look at a few partial answers to that, focusing on Frequency Modulation (FM) synthesis. Introduced in the 1970s, FM synthesis is still a popular sound design tool. The mathematics of this technique involves Fourier series, Bessel functions, and a surprising link to Kepler's equation for the gravitational two-body problem.

Time: Beijing time 10:40 am -- 11:20 am, July 4, 2023
Speaker: Zhifu Xie
Affiliation: The University of Southern Mississippi, USA. Visiting Professor of Sichuan University from July 2 to July 6.

Title: On the Uniqueness of Convex Central Configurations in the Planar 4-Body Problem

Abstract: A central configuration is a specific arrangement of masses, and a planar central configuration can lead to a homographic periodic solution. It is crucial for understanding the dynamic behavior of the N-body problem, and the question of its finiteness has been a challenge for mathematicians in the 21st century. For the planar four-body problem, its finiteness has been proven by computer-assisted proof in 2006 by Hampton and Moeckel, but there is still much to understand. One conjecture is that there exists a unique convex central configuration for any four positive masses in a given order. Many research paper has attempted this question by assuming either having some equal masses or having restrictions of the geometric shape such as a trapezoid or co-circular shape. In this talk, we provide a rigorous computer-assisted proof (CAP) of the conjecture for four masses belonging to a closed domain in the mass space. The proof employs the Krawczyk operator and the implicit function theorem. Notably, we demonstrate that the implicit function theorem can be combined with interval analysis, enabling us to estimate the size of the region where the implicit function exists and extend our findings from one mass point to its surrounding neighborhood. Such methods may be applied to general nonlinear equations. The presentation is based on the joint work with Shanzhong Sun and Peng You.

Time: Beijing time 11：30 am --12:10 pm, July 4, 2023

Speaker: Xiang Yu
Affiliation: Southwestern University of Finance and Economics. Visiting Professor of Sichuan University from July 2 to July 6.

Title: On the finiteness of four-body central configurations

Abstract: The number of central configurations in the four body problem was proved to be finite, first by Hampton and Moeckel, then by Albouy and Kaloshin, when the masses are all positive. We prove that the four-body central configurations are finite for any four nonzero masses. This is a joint work with Shuqiang Zhu.

Time: Beijing time 9:00 am -- 9:40 am, July 5, 2023 (New York time: 9:00 pm—9:40 pm, July 4th, 2023)
Speaker: Edward Belbruno
Affiliation: Professor of Mathematics, Yeshiva University, New York, New York
Senior researcher, Astrophysics Dept., Princeton Univ.

Title: Applications of the McGehee Transformation to Singularities in Astrophysics, Cosmology and Celestial Mechanics

Abstract: The McGehee regularization is an interesting way to regularize the flow of a dynamical system near a singular state by blowing up the singularity. This has been recently applied in astrophysics and cosmology to the motion of a particle about a Schwarzschild black hole (2011) and to regularization of the big bang singularity (2013, 2018), respectively. Most recently, it has been applied to motion about an asteroid (2023). These problems are described and compared.

[Refs: CQG, v28, 2011 (EB, FP); CMDA. v115, 2013, (EB); CQG, v35, 2018 (EB, BKX); CMDA , v135, 2023, (EB, MG, W-TL)]

Time: Beijing time 9:50 am -- 10:30 am, July 5, 2023 (Mexico City time: 7:50 pm – 8:30 pm, July 4rd, 2023)
Speaker: Ernesto Pérez-Chavela
Affiliation: Instituto Tecnológico Autónomo de México (ITAM), México

Title: Relative equilibria on the sphere (Joint work with Toshiaki Fujiwara).

Abstract: The simplest solutions of the N–body problem are those where the mutual distances among the masses remain constant for all time, that is, each mass rotates uniformly through the center of mass of the system, the motions be- have as if they belong to a rigid body. For N = 3 on the Euclidean space there are exactly five relative equilibria: three collinear (Euler relative equilibria) and two planar forming an equilateral

triangle (Lagrange relative equilibria). The big difficulty to study relative equilibria on the sphere S2, that we call RE by short, is the absence of the center of mass as a first integral, since many of the standard methods used in the classical case don't apply any more. Without the center of mass we do not know how to determine the rotation axis.

In this talk I will show a new geometrical method to study RE on the sphere, when the masses are moving under the influence of a general potential which only depends on the mutual distances among the masses. First we prove the existence of two new integrals of motion, which can be seen as an extension of the center of mass. These two new integrals allow us determine the rotation axis. We restrict our analysis to the case N = 3. Applying our method, we give some new families of Euler and Lagrange RE on the sphere for the cotangent potential (the natural extension of the Newtonian potential to the sphere).

Time: Beijing time 10:40 am --11:20 am, July 5, 2023
Speaker: Alexander Wires
Affiliation: Visiting Professor of Sichuan University from July 1 to September 1.

Title: Deconstruction/Reconstruction of Extensions

Abstract: For a fixed signature, we consider the cohomological machinery regarding the decomposition and reconstruction of extensions realizing affine datum in arbitrary varieties of universal algebras; very loosely speaking, it can be said that these are extensions in which there exists a total ternary operation on the algebra which effects a compatible structure of module polynomials on the fibers of the extension map and explicitly determines the semidirect product associated to the congruence on the kernel of the extension map which is generated by the diagonal.

The resulting low-dimensional abelian cohomology groups associated to extensions is a far-reaching generalization to arbitrary varieties of universal algebras of the classical development for groups, non-associative algebras and various multilinear algebras which have only recently been investigated. The machinery is replete with items not often found in general approaches to cohomology such as a notion of principal derivation connected to stabilizing automorphisms and a satisfaction relation for identities in the partial structure associated to the mixed action and 2-cocycle signature. The cohomology group accepts an additional parameter which controls the equational theories of possible extensions of the datum. This is encapsulated in a Galois connection between the lattice of varieties containing fixed datum and cohomology. In varieties with a weak-difference term, the Galois connection witnesses the classical subgroup inclusion of abelien extensions in cohomology. A general Well's-type exact sequence for automorphisms of extensions of affine datum is given.

Time: Beijing time 11:30 am -- 12:10 pm, July 5, 2023
Speaker: Zhifu Xie
Affiliation: The University of Southern Mississippi, USA. Visiting Professor of Sichuan University from July 2 to July 6.

Title: Interval Analysis, Krawczyk Operator and SageMath

Abstract: Interval analysis has a long history but its numerical computation was implemented in computer recently. In this talk, we first give a brief introduction of the interval arithmetic and the Krawczyk operator (the extension of Newtwon's method). Then we introduce how to use SageMath to implement interval arithmetic computations. Finally an example is presented to solve a system of nonlinear algebraic equations by using interval computations and the Krawczyk operator.